Lecture
on

Mechanics of Solids

Prepared by

Anisul Islam

Lecturer
Department of ME, UGV

BASIC COURSE INFORMATION

Course Title	Mechanics of Solids
Course Code	ME 0715-2243
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	4th Semester

Mechanics of Solids-I

Course Code: ME 0715-2243	**CREDIT:03**
Course Teacher: Anisul Islam	**TOTAL MARKS:150**
Mid Exam Hours 2	**CIE MARKS: 90**
Semester End Exam Hours 3	**SEE MARKS: 60**

Course Learning Outcomes (CLOs): After completing this course successfully, the students will be able to-

CLO 1	**Understand** the types of loads and stresses in different loaded members and develop skills to determine them.
CLO 2	**Identify** the magnitude of safe loads and stresses to operate individual members and structures without failure.
CLO 3	**Perform** structural design by determining the value of shear force and bending moment at a given point for structural members such as a beam, column, frame, etc.
CLO 4	**Assess** the deflections and deformations of loaded flexural members.

SL	Content of Course	Hrs	CLOs
1	Introduction and analysis of internal forces: Tension, compression, axial stress, strain, shear stress, shear force, bending moment, deformation, stress-strain diagram, elasticity, rigidity, yield strength, ultimate strength, strain hardening, strain softening, elastic limits, plastic limit, fracture point, ductility, engineering stress-strain, true stress-strain, bone shear failure, hardness, brittleness, resilience and toughness.	8	CLO1, CLO 2
2	Definition of some mechanical properties of materials: Poisson's ratio, definition of torque and torsion, stresses in thin walled pressure vessels: hoop and longitudinal stress, Thermal Stress-strain Equation, Problem Solving, Thin Walled Pressure Vessel Equation	8	CLO2, CLO 3
3	Shear Force and Bending Moment Diagram, Bending Stress, Equation, Problem Solving	6	CLO2, CLO3
4	Torsion and Mohr's Circle	10	CLO3, CLO4

Text Book:

Strength of Materials (4th ed.). Pytel, A. & Singer, F. L., Harper Collins Inc., 1987; ISBN: 0-06-350599-1
Mechanics of Materials. Beer and Johnston; McGraw- Hill, 2009; ISBN: 0073529389
Mechanics of Materials (9th ed.). Hibbeler, R. C., Pearson Prentice Hall, 2014; ISBN: 10: 0-13-3254429

ASSESSMENT PATTERN
CIE- Continuous Internal Evaluation (90 Marks)
SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	**10**
Understand	**10**
Apply	**10**
Analyze	**15**
Evaluate	**10**
Create	**5**

Bloom's Category Marks (out of 90)	Tests (45)	Assignments (15)	Quizzes (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10		10	Attendance 15
Understand	5		5	
Apply	10			
Analyze	10			
Evaluate	5			
Create	5	15		

Course plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Topic	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Tension, compression, axial stress, strain, Shear stress, shear force, bending moment, deformation	Lecture, discussion, group work	Quiz, Written Exam	CLO1
2	Tensile stress-strain diagram	Oral Presentation, Lecture, discussion, group work	Assignment, Written, Quiz	CLO1
3	Elasticity, rigidity, yield strength, ultimate strength, Elastic limits, plastic limit and fracture point, Strain hardening and strain softening properties	Video lecture	Report writing, Demonstration	CLO1
4	Details of ductility, True stress-strain Diagram, Bone shear failure	Lecture	Viva, Quiz	CLO1, CLO2
5	Hardness and brittleness, Resilience and toughness, Poisson’s ratio and torque	Lecture, discussion, group work	Project, Field visit	CLO2
6	hoop stress and longitudinal stress	Discussion, Video Presentation	Quiz, Written Exam	CLO2
7	SFD and BMD Problem solving	Case-based Learning, Demonstration	Assignment, Written, Quiz	CLO2
8	Thin Walled Pressure Vessel Equation, Problem Solving	Lecture, discussion, group work	Report writing, Demonstration	CLO2
9	Thermal Stress-strain Equation	Oral Presentation, debate	Viva, Quiz	CLO2
10	Expression for bending Stress	Video lecture	Project, Field visit	CLO3

Course plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Topic	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
11	Bending stress related problem	Lecture, Oral Presentation	Quiz, Written Exam	CLO3
12	Torsion	Lecture, discussion,	Assignment, Written, Quiz	CLO3
13	Torsion	Discussion, Video Presentation	Report writing, Demonstration	CLO 3
14	Mohr Circle construction Principle	Case-based Learning, Demonstration	Viva, Quiz	CLO3
15	Mohr Circle Related Problem	Lecture, discussion, group work	Project, Field visit	CLO4
16	Mohr Circle Related Problem	Oral Presentation	Quiz, Written Exam	CLO4
17	Practice and Exercise, Practice and Exercise, Practice and Exercise	Group Discussion	Assignment, Written, Quiz	CLO3, CLO4

Basic Materials Properties

Week 1

Pages (9-16)

Tension and Compression

Tension and compression are forces that act on materials, causing them to stretch or squeeze in different ways:

Tension
A pulling or stretching force that causes a material to elongate or deform. Tension is often referred to as positive stress.

Compression
A pushing or squeezing force that causes a material to shorten or deform. Compression is often referred to as negative stress.

Axial Stress and Strain

Stress can be defined by ratio of the perpendicular force applied to a specimen divided by its original cross sectional area, formally called engineering stress. To compare specimens of different sizes, the load is calculated per unit area, also called normalization to the area. Force divided by area is called stress. In tension and compression tests, the relevant area is that perpendicular to the force. In shear or torsion tests, the area is perpendicular to the axis of rotation. The stress is obtained by dividing the load (F) by the original area of the cross section of the specimen (A_0).

$$\sigma = \frac{F}{A_0}$$

Axial Stress and Strain

Strain is the ratio of change in length due to deformation to the original length of the specimen, formally called engineering strain. strain is unit less, but often units of m/m (or mm/mm) are used.

The strain used for the engineering stress-strain curve is the average linear strain, which is obtained by dividing the elongation of the gage length of the specimen, by its original length.

$$\varepsilon = \frac{l_i - l_o}{l_o} = \frac{\Delta l}{l_o}$$

Shear Stress

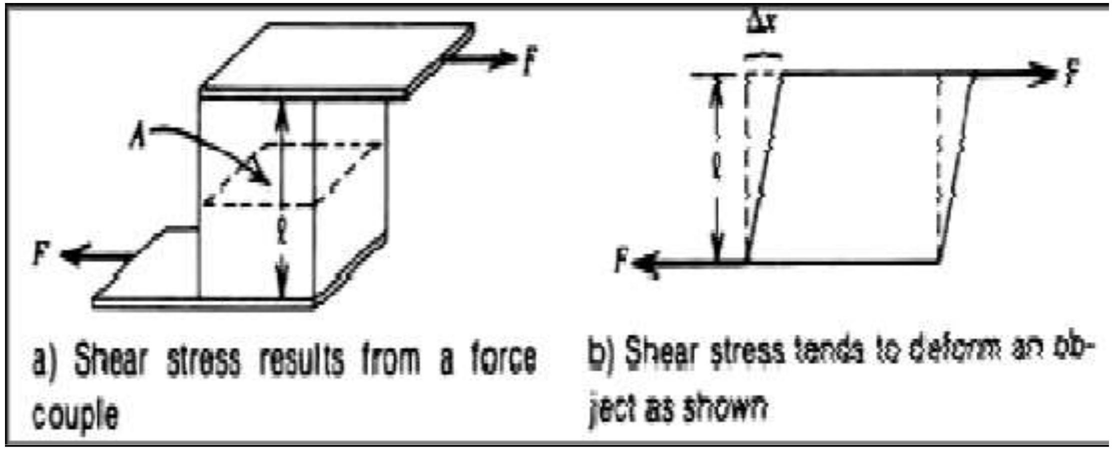
Tensile and compressional stress can be defined in terms of forces applied to a uniform rod.

Shear stress is defined in terms of a couple that tends to deform a joining member.

$$\tau = \frac{F}{A}$$

where

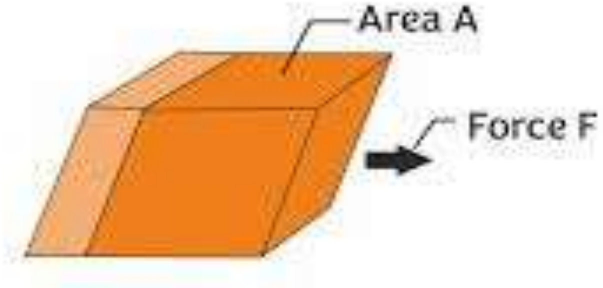
- τ shear stress [Pa]
- F applied force [N]
- A cross-sectional area [m^2]



a) Shear stress results from a force couple

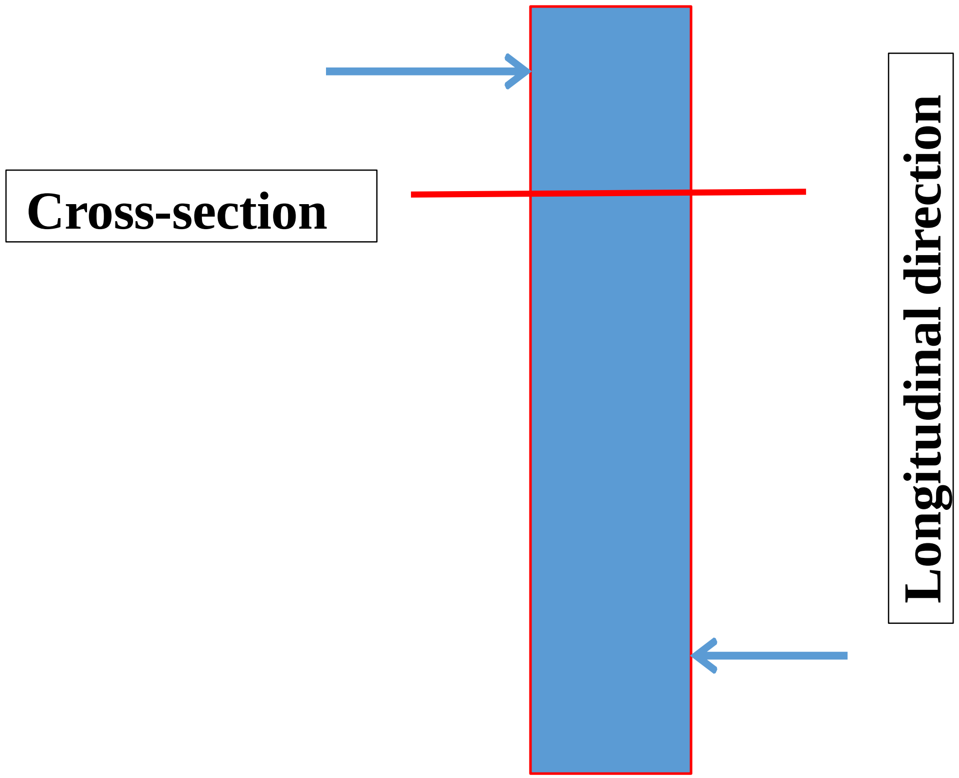
b) Shear stress tends to deform an object as shown

Shearing Stress



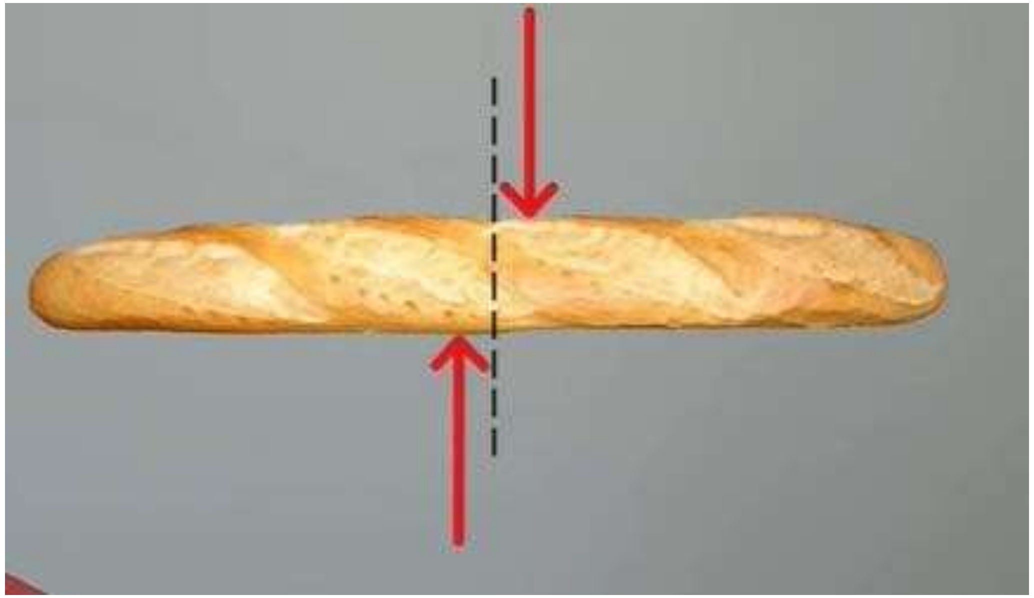
Shear force definition

Shear force is a force acting in a direction that's parallel to a surface or cross section of a body. The word shear in the term is a reference to the fact that such a force can cut, or shear, through the surface or object under strain. In solid mechanics, shearing forces are unaligned forces pushing one part of a body in one specific direction, and another part of the body in the opposite direction. When the forces are collinear (aligned into each other), they are called compression forces.



Shear force example

Example: Cutting bread, Cutting paper, beam bending, drawing picture, sliding cash note, painting, brushing teeth, bone shear failure etc.



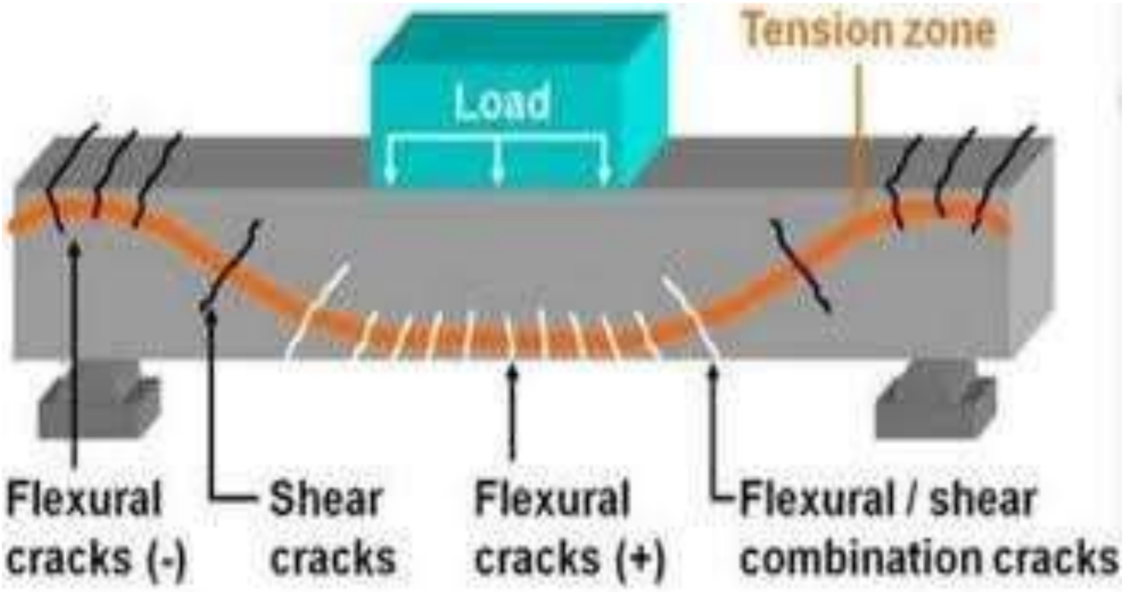


Figure: Shear force example

Bending Moment

Bending moment is the internal resistance of a structural element to bending when an external force is applied. It's a measure of the effect of a load that causes a structural member to bend.

Bending moment = Force x Distance

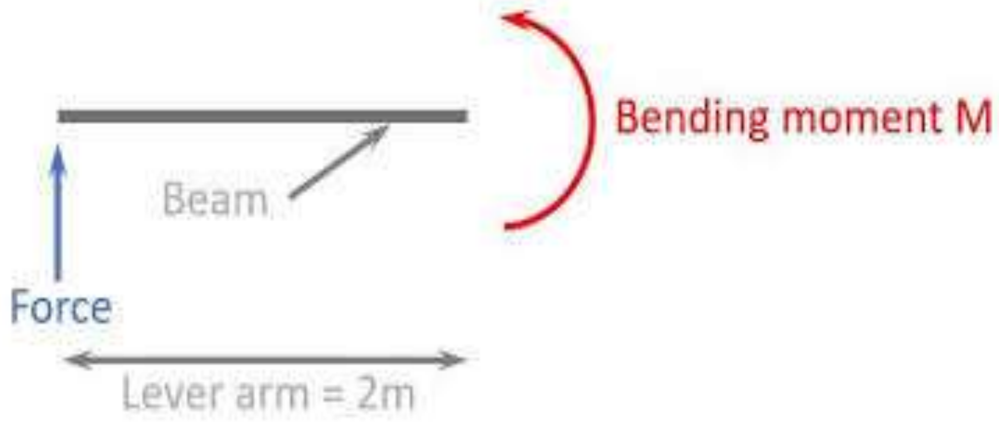


Figure: Bending moment

Deformation

In civil engineering, **deformation** is the change in shape or size of an object due to an applied force or change in temperature.

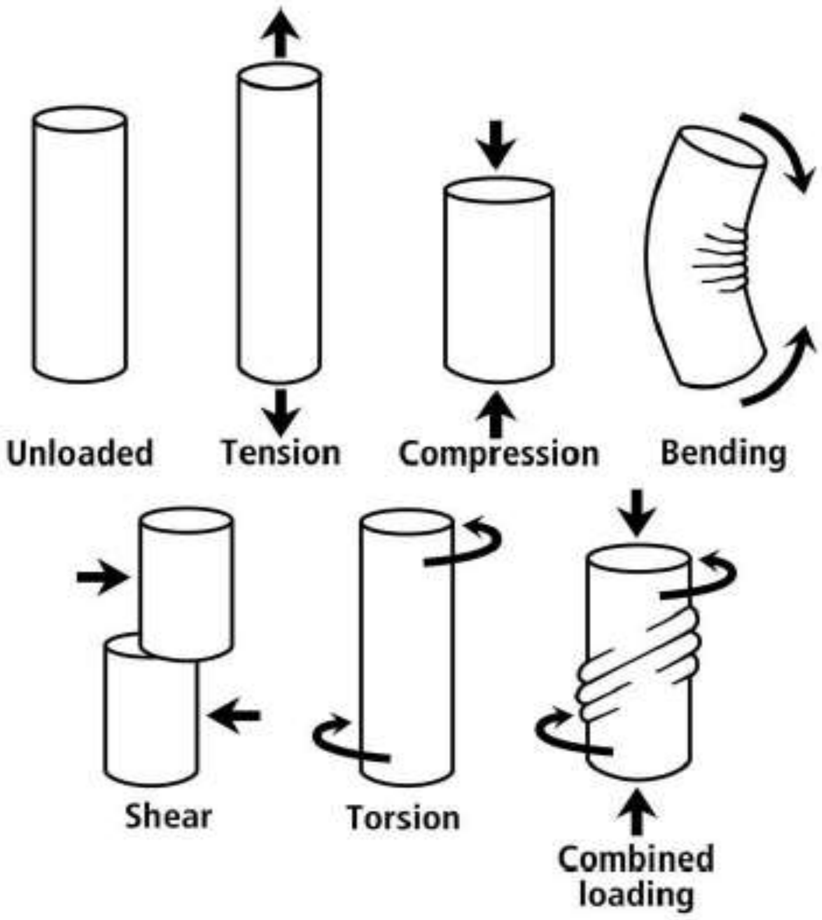


Figure: Different types of loading condition in material.

Basic Materials Properties

Week 2

Pages (17-18)

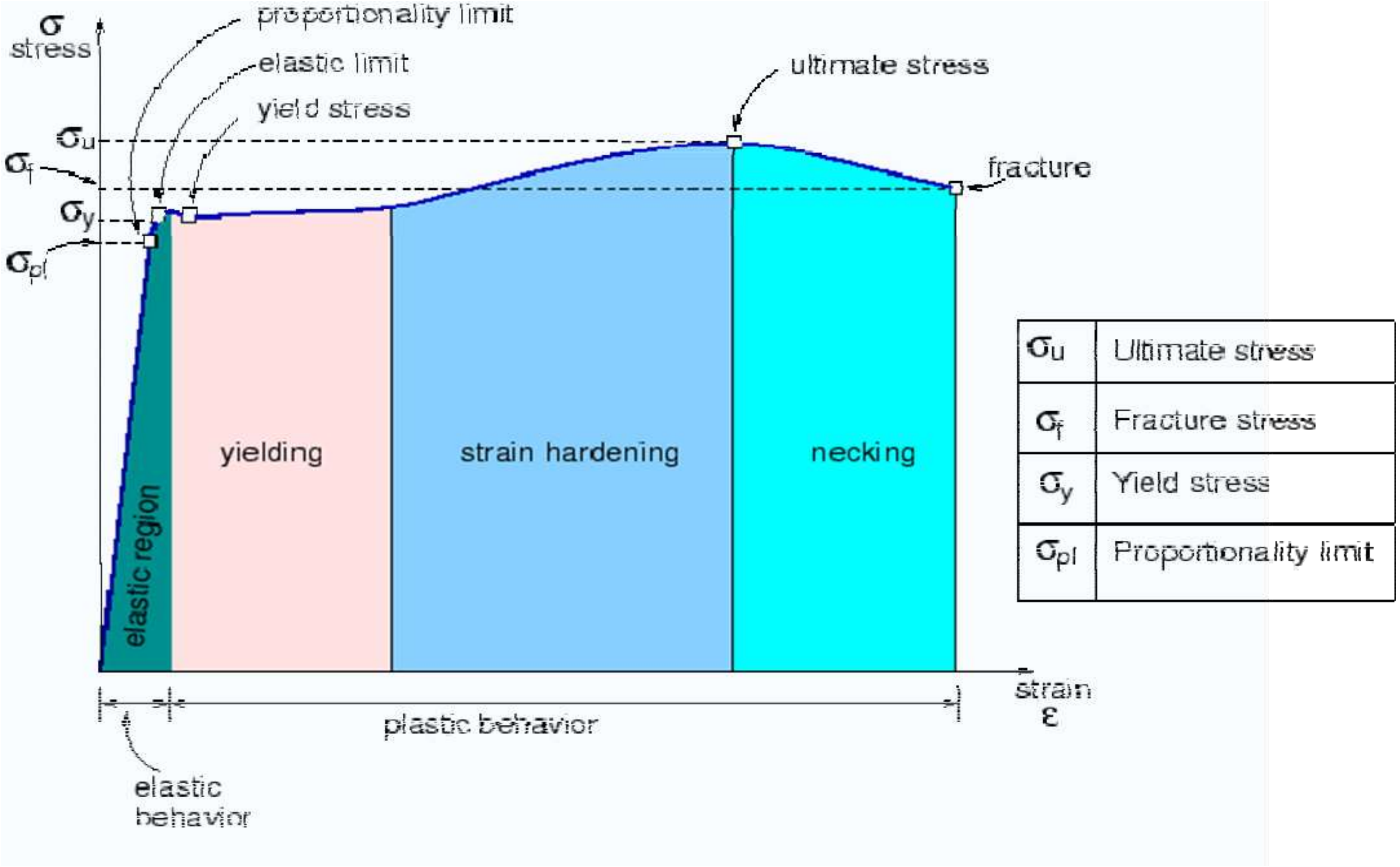


Figure: Tensile stress-strain diagram of reinforcement.

Basic Materials Properties

Week 3

Pages (20-23)

A tensile stress-strain diagram, also known as a stress-strain curve, illustrates the relationship between a material's stress and strain. The curve can be used to determine a material's characteristic values, such as its elastic behavior and tensile strength.

Some properties of a tensile stress-strain diagram include:

Proportional limit: The highest stress at which stress and strain are directly proportional. This is the point on the curve where the deviation from the straight-line portion begins.

Elastic limit: The greatest stress a material can withstand without any permanent strain.

Yield strength: The stress required to produce a small amount of plastic deformation.

Modulus of resilience: The work done on a unit volume of material as the force increases from zero to the elastic limit. It is also known as Young's modulus of elasticity, and is analogous to the stiffness of a spring.

Hooke’s Law

For materials stressed in tension, at relatively low levels, stress and strain are proportional through:

$$\sigma = E\varepsilon$$

Here, constant E is known as the modulus of elasticity, or Young’s modulus.

Within the linear region, a specific type of material will always follow the same curves despite different physical dimensions. Thus, it can say that the linearity and slope are a constant of the type of material only. In tensile and compressional stress, this constant is called the *modulus of elasticity* or *Young's modulus (E). So*, $E = \frac{F/A}{\Delta l/l}$

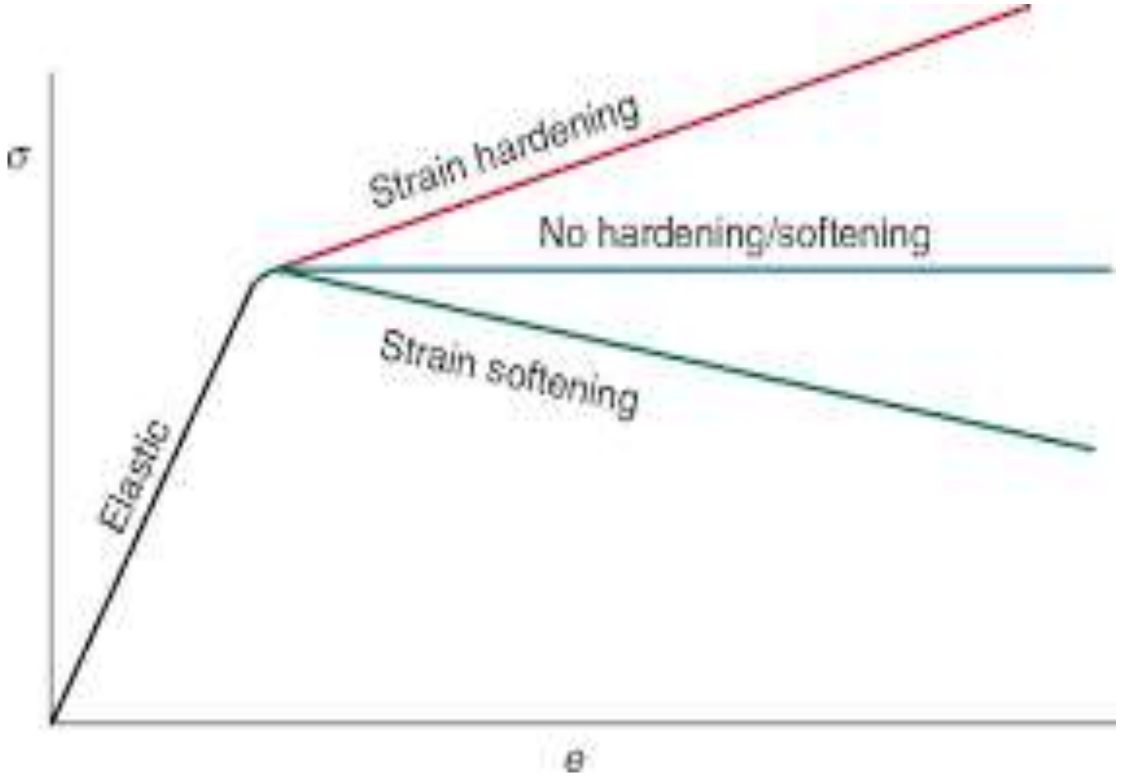
where stress = F/A in N/m^2 strain = Dl/l unit less

E = Modulus of elasticity in N/m^2

The modulus of elasticity has units of stress, that is, N/m^2. The following table gives the modulus of elasticity for several materials. In an exactly similar fashion, the shear modulus is defined for shear stress-strain as modulus of elasticity.

Material	Modulus (N/m^2)
Aluminum Copper Steel	6.89×10^{10} 11.73 X 10'° 20.70 X 10^{10} 2.1×10^8

Strain hardening refers to a material becoming stronger and harder with increasing strain, while strain softening means a material becomes weaker with increasing strain; essentially, in **strain hardening**, stress increases with deformation, whereas in **strain softening**, stress decreases with deformation as strain increases.



Basic Materials Properties

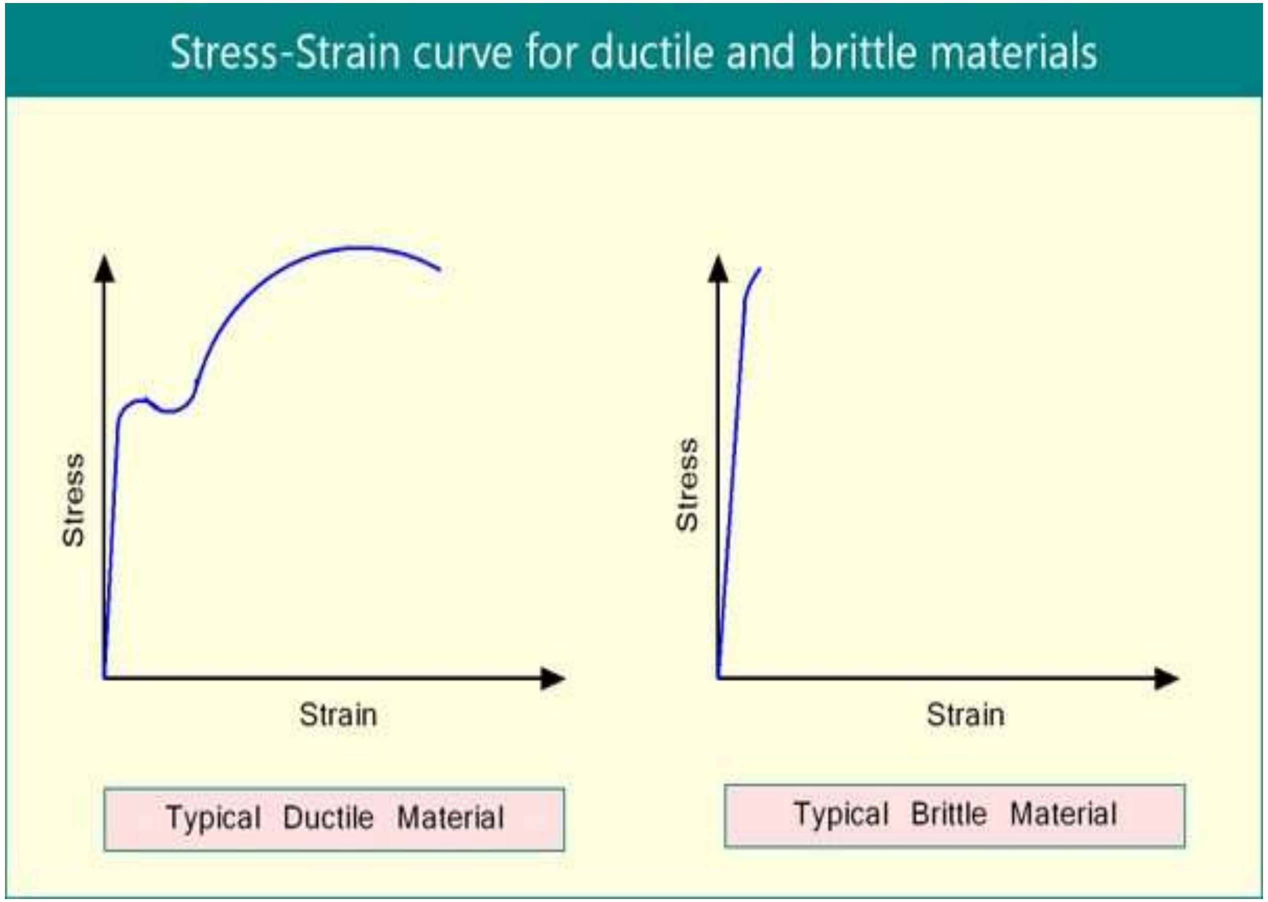
Week 4

Pages (25-28)

Ductility:

Ductility describes the extent to which a material (or structure) can undergo large deformations before fracture. The term is used in earthquake engineering to designate how well a building will endure large lateral displacements imposed by ground shaking.

A ductile structure's ability to dissipate energy during an earthquake is, therefore, also advantageous as it will keep deforming without reaching ultimate failure or collapse. An example of a ductile structure is a properly detailed steel frame with that will enable it to undergo large deformations before the onset of failure.



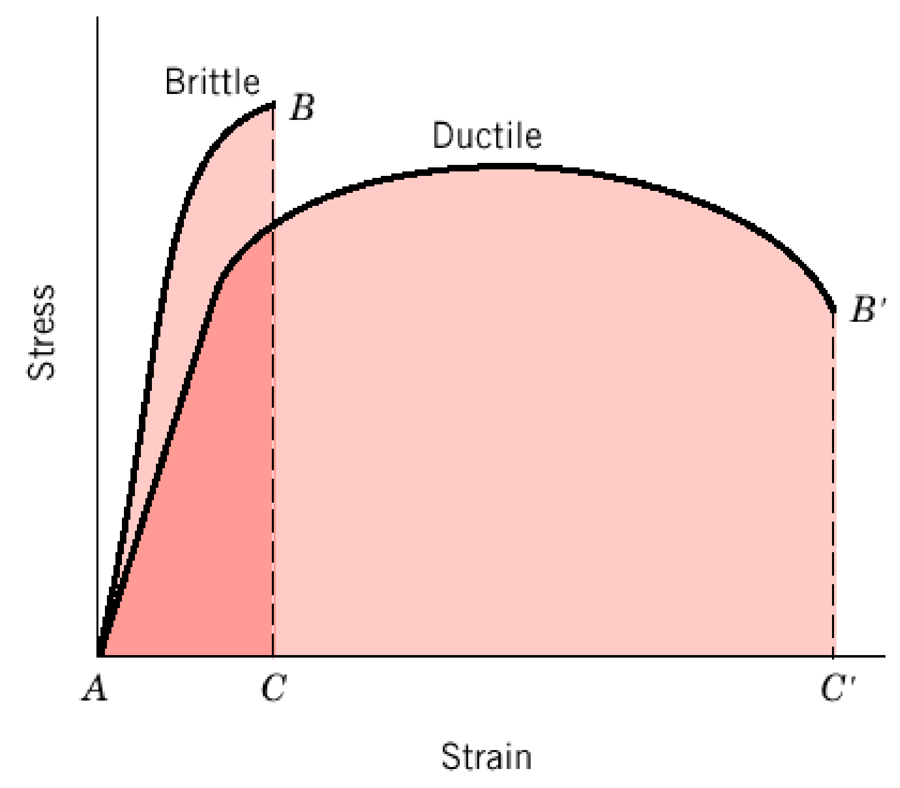


Figure: Brittle vs Ductile Material.

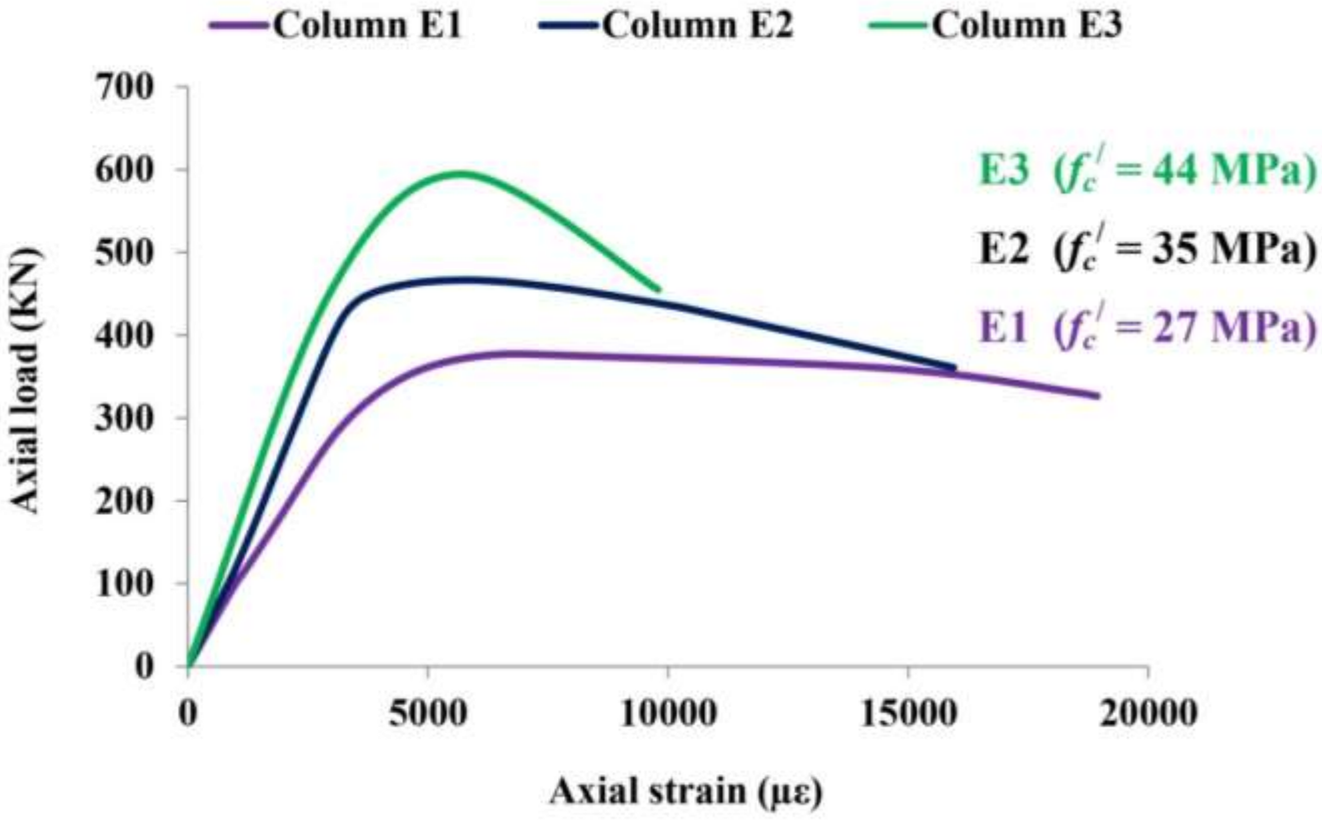
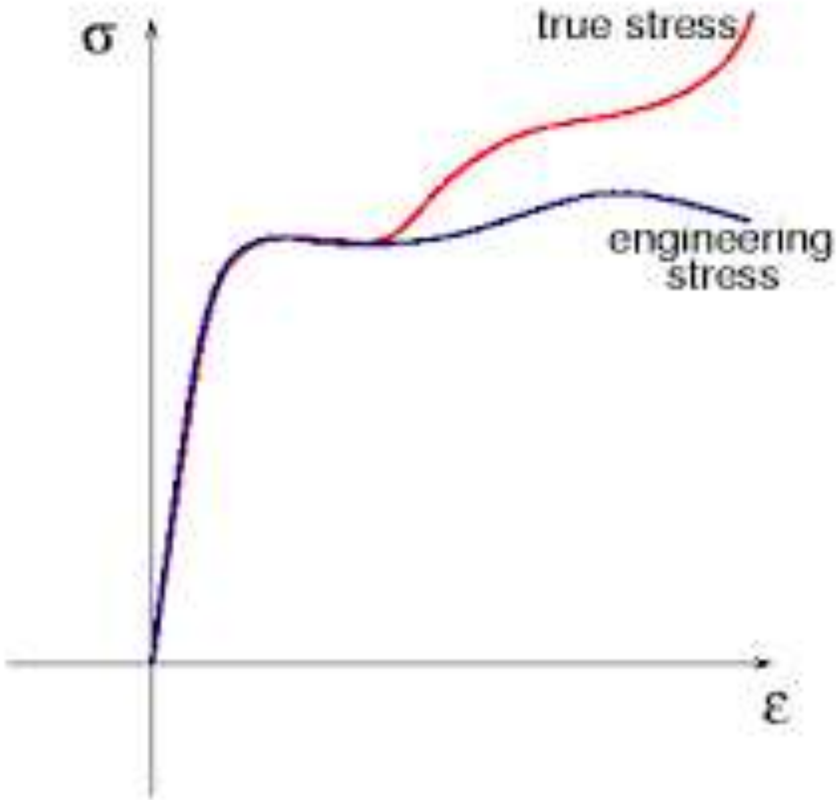


Figure: What do you think which colour exhibits less ductile behaviour?????.

True Stress versus Engineering Stress

While both "stress" and "strain" measure deformation in a material, the key difference between "**true stress/strain**" and "**engineering stress/strain**" lies in how the cross-sectional area is considered: "engineering" calculations use the original cross-sectional area of the material, while "true" calculations use the instantaneous, changing cross-sectional area during deformation, making true stress/strain values always higher than engineering stress/strain, especially at large deformations.



Basic Materials Properties

Week 5

Pages (30-33)

Hardness

Hardness is the resistance of a material to localized plastic deformation. Hardness is a measure of a material's resistance to permanent deformation. Toughness is a measure of how much deformation a solid material can undergo before fracturing. Hardness is often inversely related to ductility, so the ductile metals mentioned above typically have relatively low hardness.

The product EI is termed the "beam stiffness", or sometimes the "flexural rigidity". It is often given the symbol Σ. It is a measure of how strongly the beam resists deflection under bending moments.

Stiffness is how a component resists elastic deformation when a load is applied. Hardness is resistance to localized surface deformation.

The modulus of rigidity, also known as shear modulus, is defined as the ratio of shear stress to shear strain of a structural member. This property depends on the material of the member: the more elastic the member, the higher the modulus of rigidity.

Stiffness is the extent to which an object resists deformation in response to an applied force.

Elastic modulus is a measure of stiffness. **Steel is stiffer than rubber thus its modulus is higher**.

Stiff → not easily bent or changed in shape. Rigid → unable to bend or be forced out of shape, not flexible.

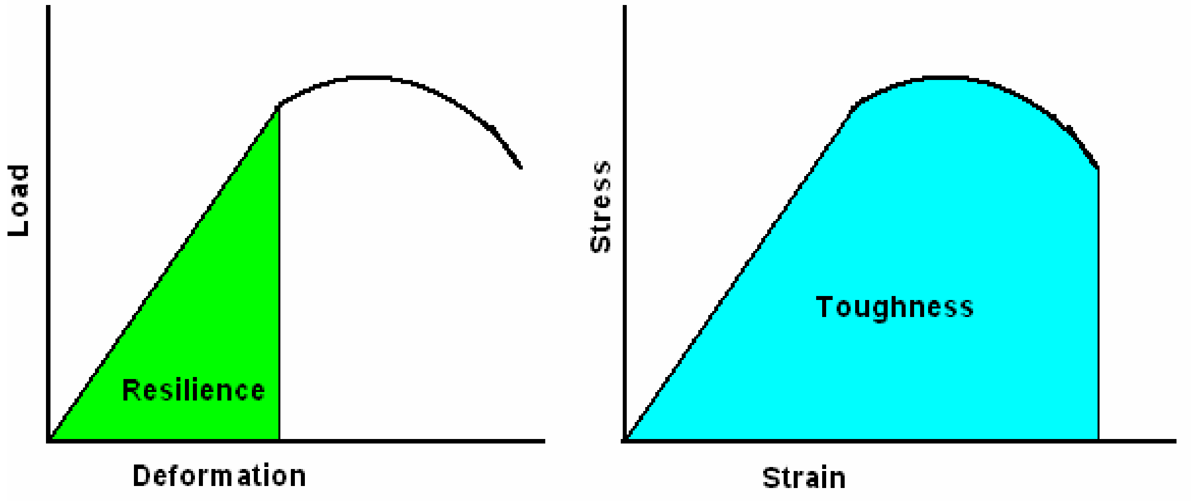
Rigidity, on the other hand, refers to the opposite property of elasticity. It is the inability of a material to deform under an external force. Flexibility is the ability of a material to bend, twist or stretch without breaking. This property depends on the material's elasticity and plasticity.

Resilience

The resilience of the material is the triangular area underneath the elastic region of the curve. In physics and engineering, resilience is defined as the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading to have this energy recovered.

Toughness

The area underneath the stress-strain curve is the toughness of the material- i.e. the energy the material can absorb prior to rupture.. It also can be defined as the resistance of a material to crack propagation. The ability of a metal to deform plastically and to absorb energy in the process before fracture is termed toughness. A material with high strength and high ductility will have more toughness than a material with low strength and high ductility.



Comparison between resilience and toughness of metals

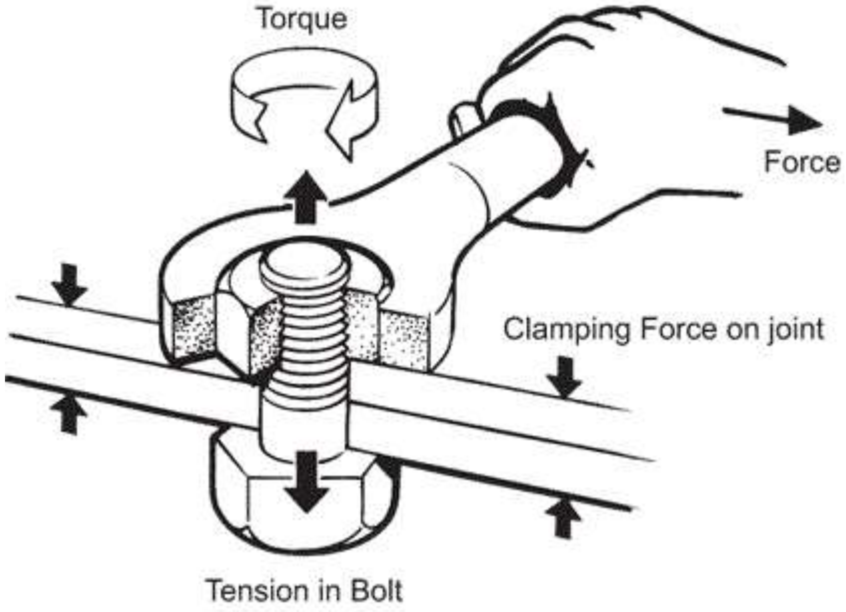
Poisson ratio

Poisson ratio is the ratio of transverse contraction (or expansion) strain to longitudinal extension strain in the direction of stretching force. Tensile deformation is considered positive and compressive deformation is considered negative.

$$v = -\frac{d\varepsilon_{trans}}{d\varepsilon_{axial}}$$

Torque (Twisting Moment)

Torque typically refers to the tendency of a force to rotate an object around an axis. It is commonly used in the context of engines, vehicles, and rotating machinery. Torque is a measure of the rotational force applied to an object and is typically expressed in units such as newton-meters or foot-pounds.



Basic Materials Properties

Week 6

Pages (35-36)

Hoop stress and longitudinal stress are two types of stress that occur in cylindrical pipes:

•**Hoop stress**

Also known as tangential stress, this stress runs around the pipe in a ring-like pattern. It's caused by internal pressure and acts perpendicular to the pipe's axis and radius.

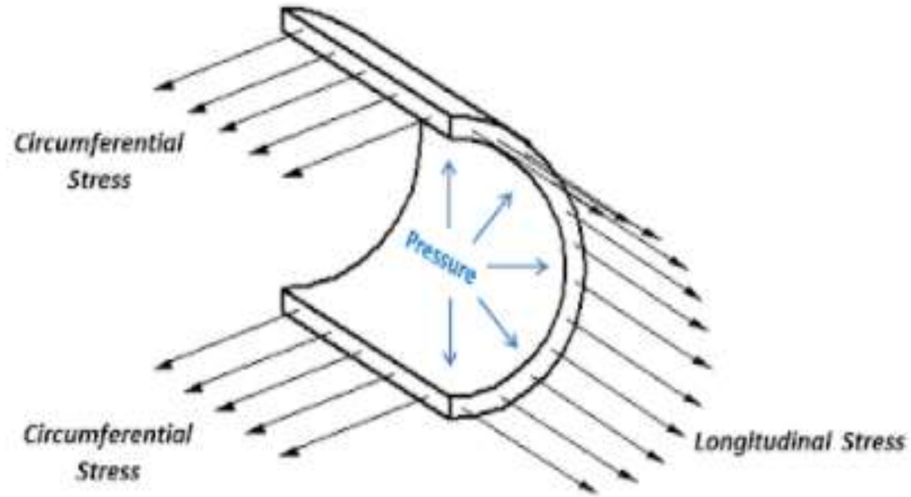
•**Longitudinal stress**

This stress runs up the pipe in an axial direction. It occurs when the length of the pipe changes due to normal stress.

The relationship between hoop stress and longitudinal stress depends on the thickness of the pipe wall relative to its radius:

•**Thin-walled pipes**

In most practical engineering applications, the hoop stress is about twice the magnitude of the longitudinal stress. This is because the wall thickness is small compared to the radius..



Longitudinal stress and Hoop stress

Thick-walled pipes

In these cases, the relationship between the two types of stress is more complex and may require more advanced analysis.

Click this link

Hoop Stress

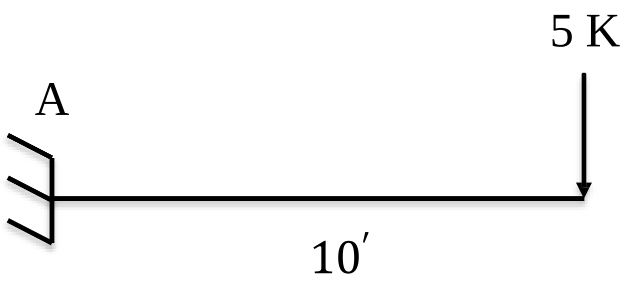
Longitudinal Stress

SFD & BMD Problem Solving

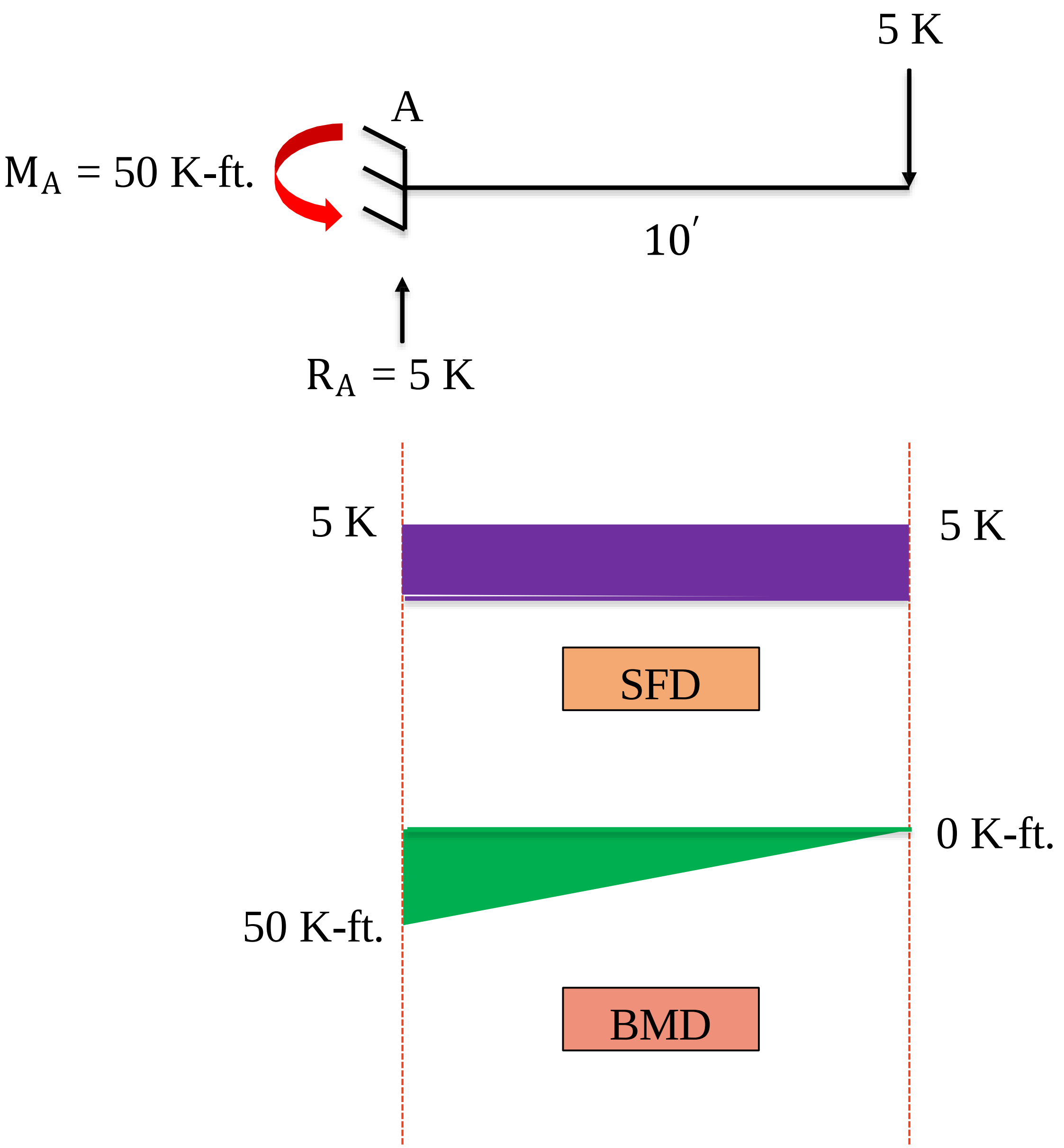
Week 7

Pages (38-59)

Problem-1: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$\sum F_y = 0$

$\Rightarrow 5 - R_A = 0$

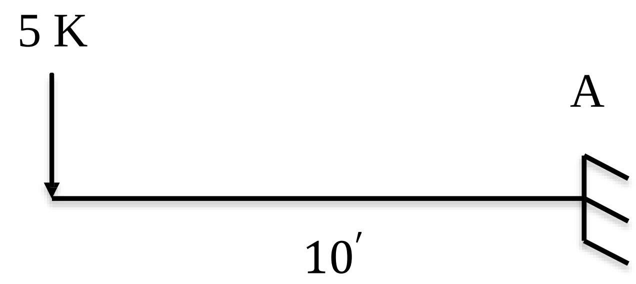
$\Rightarrow R_A = 5$ K.

$\sum M_A = 0$

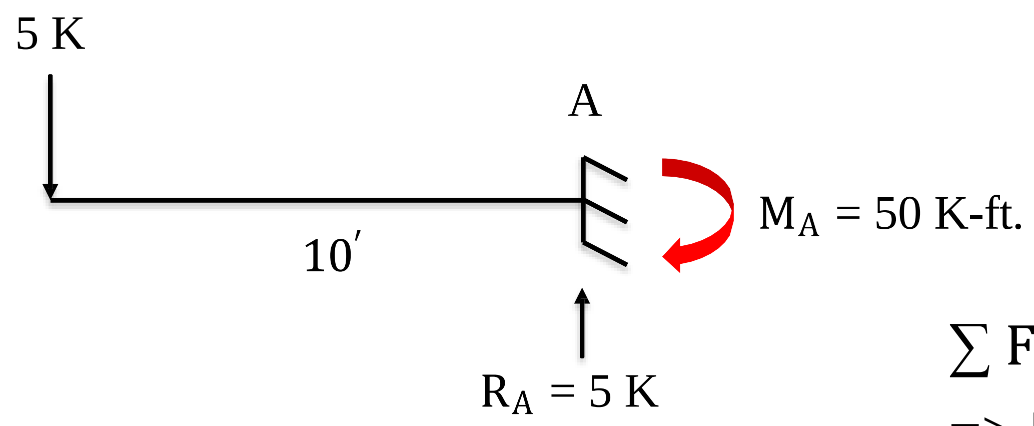
$\Rightarrow 5 \times 10 - M_A = 0$

$\Rightarrow M_A = 50$ K-ft.

Problem-2: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



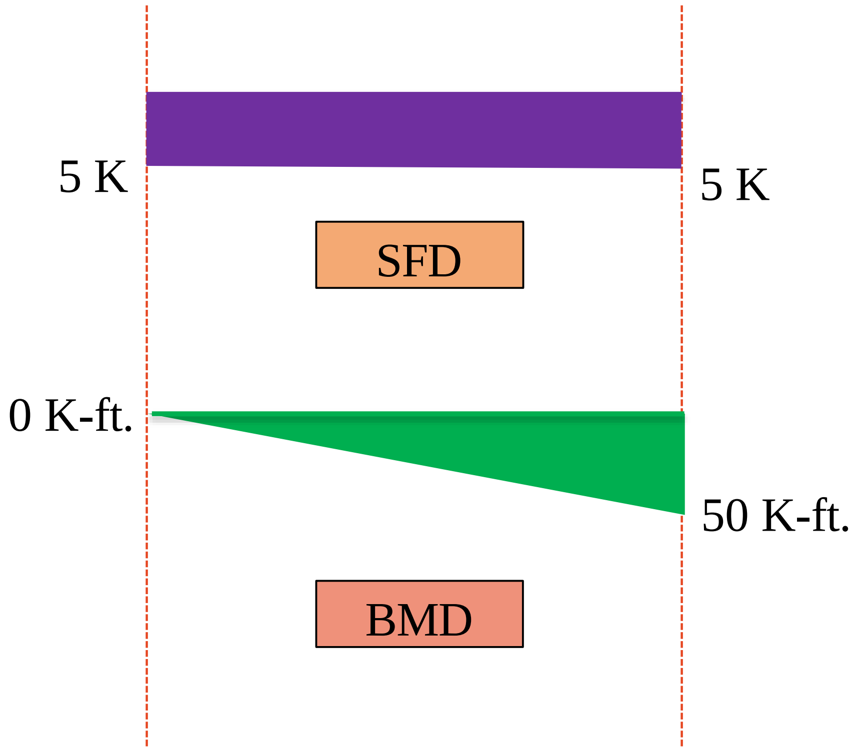
Solution:



$$\sum F_y = 0$$

$$\Rightarrow 5 - R_A = 0$$

$$\Rightarrow R_A = 5\text{ K.}$$

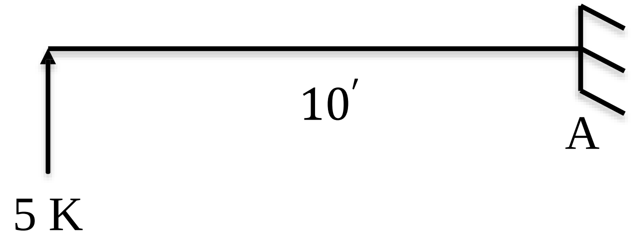


$$\sum M_A = 0$$

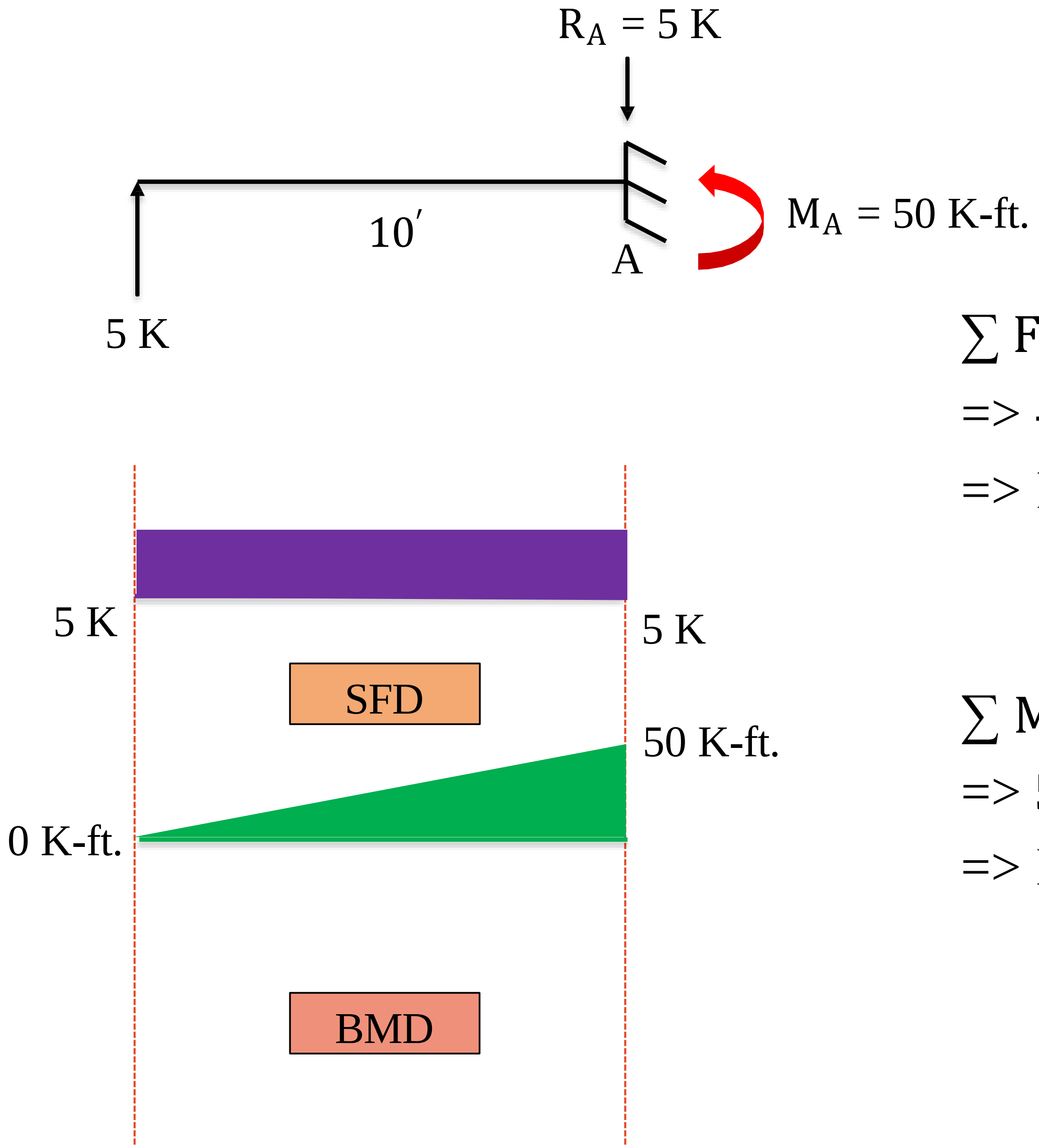
$$\Rightarrow 5 \times 10 - M_A = 0$$

$$\Rightarrow M_A = 50\text{ K-ft.}$$

Problem-3: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$\sum F_y = 0$

$=> - 5 + R_A = 0$

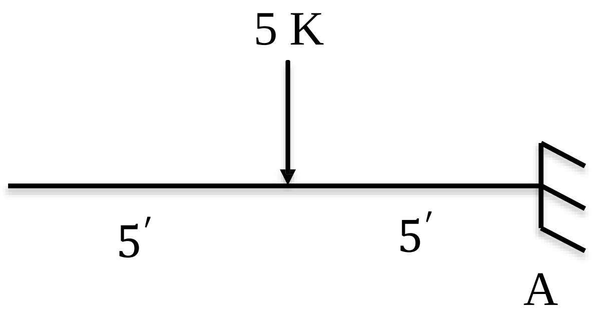
$=> R_A = 5$ K.

$\sum M_A = 0$

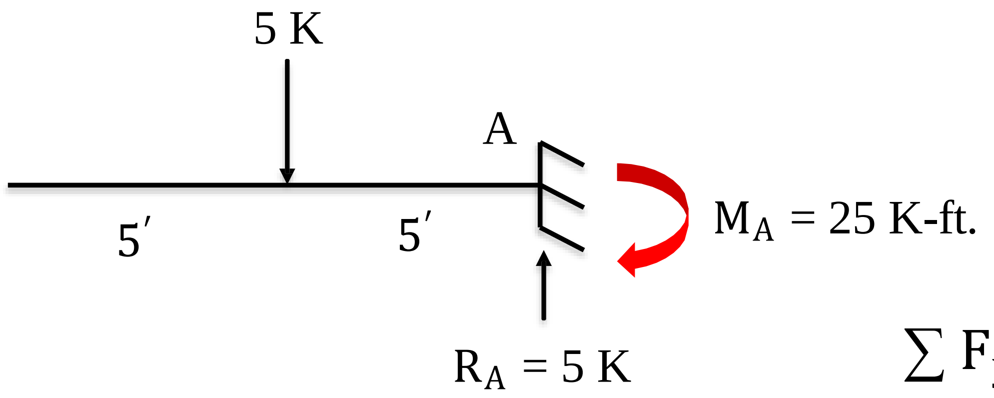
$=> 5 \text{ x } 10 - M_A = 0$

$=> M_A = 50$ K-ft.

Problem-4: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



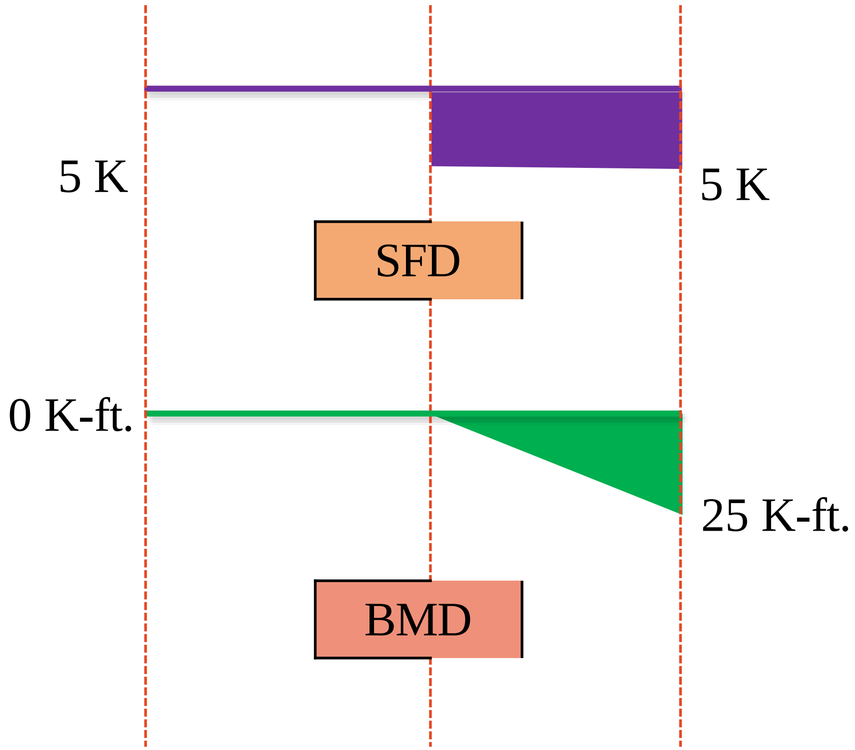
Solution:



$\sum F_y = 0$

$=> 5 - R_A = 0$

$=> R_A = 5$ K.

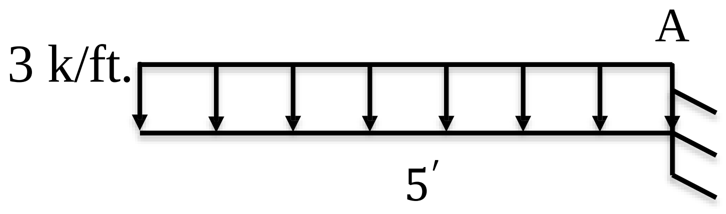


$\sum M_A = 0$

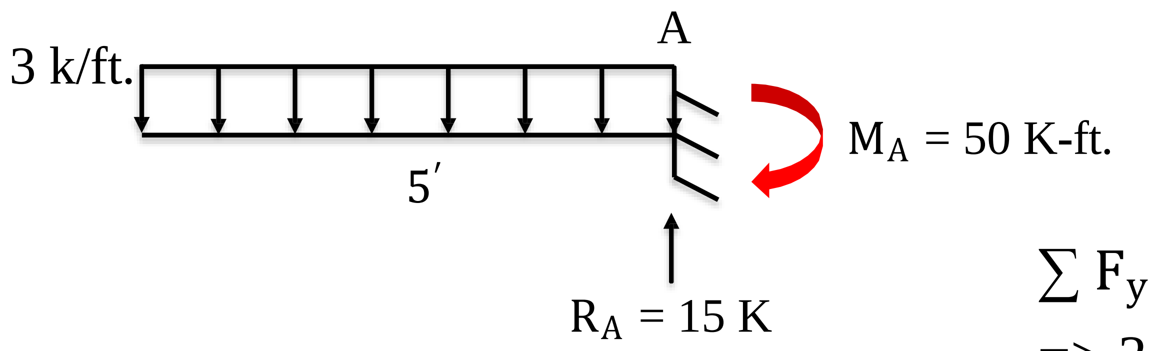
$=> -5 \times 5 + M_A = 0$

$=> M_A = 25$ K-ft.

Problem-5: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



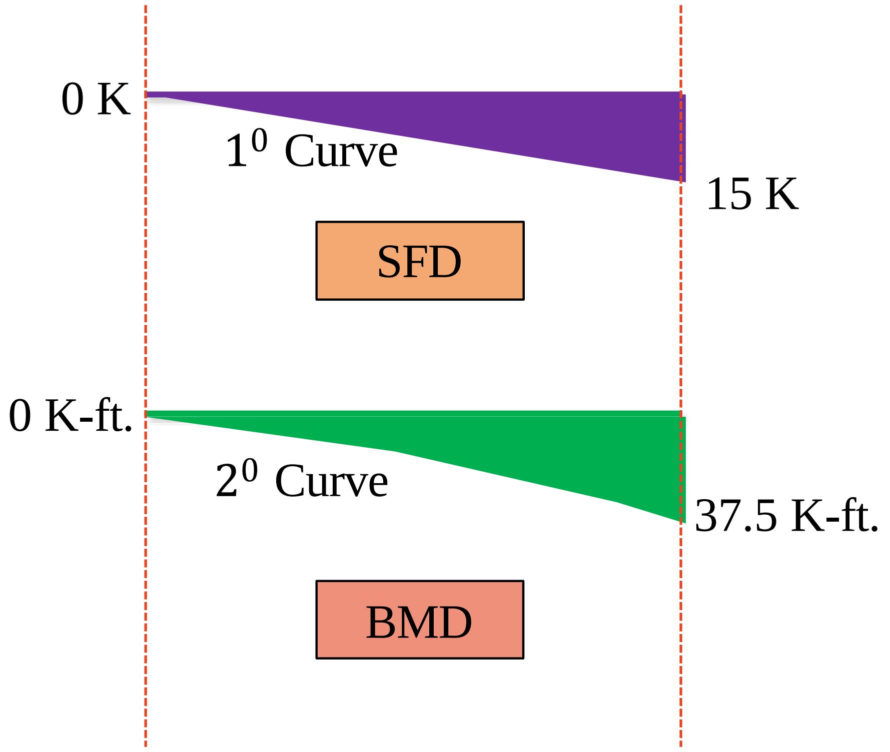
Solution:



$\sum F_y = 0$

$\Rightarrow 3 \text{ x } 5 - R_A = 0$

$\Rightarrow R_A = 15$ K.

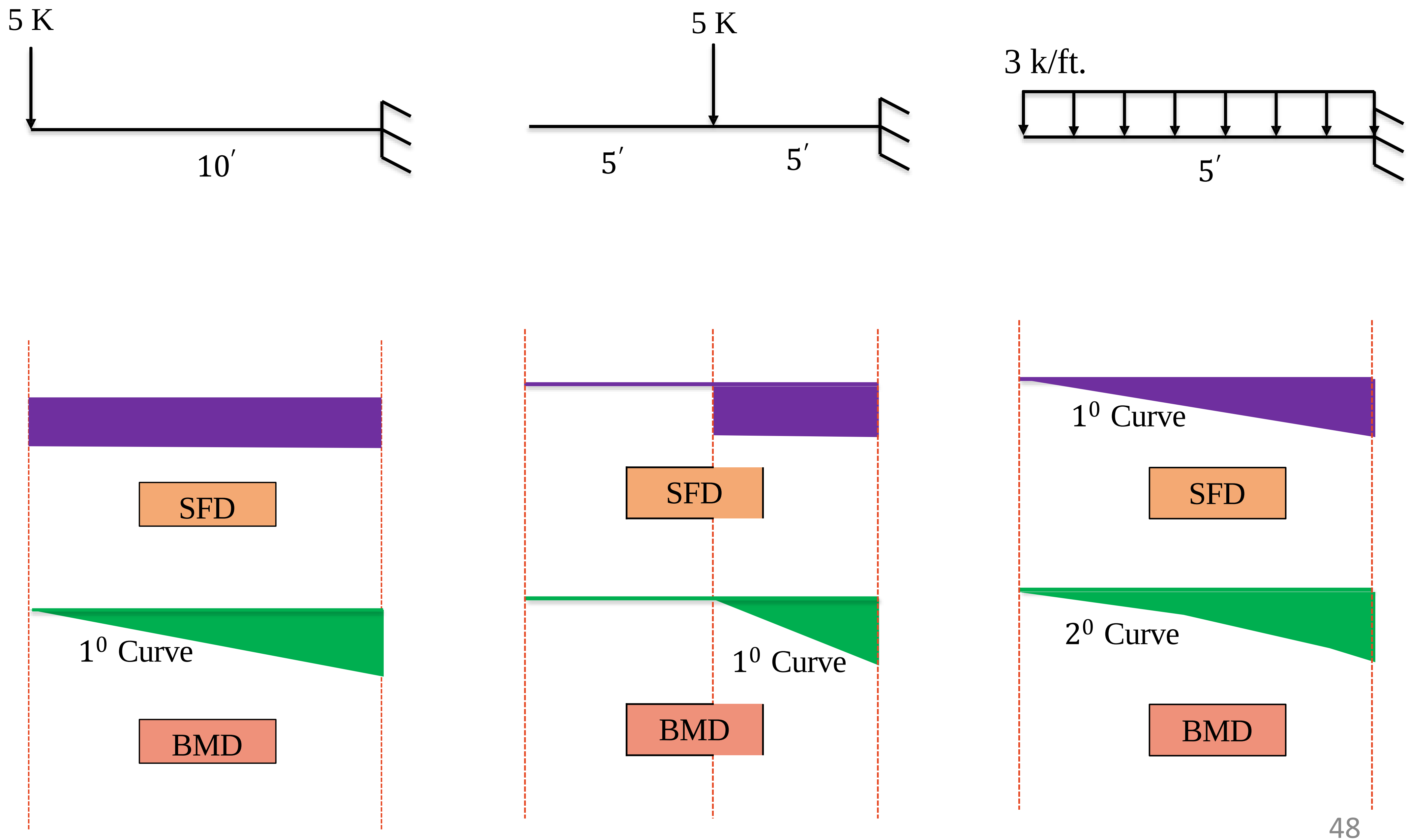


$\sum M_A = 0$

$\Rightarrow \frac{wl^2}{2} - M_A = 0$

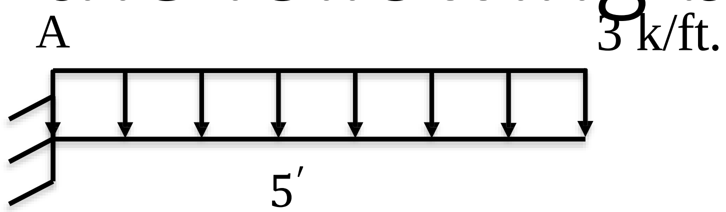
$\Rightarrow M_A = \frac{3 \; x \; 5^2}{2} = 37.5$ K-ft.

Solution:

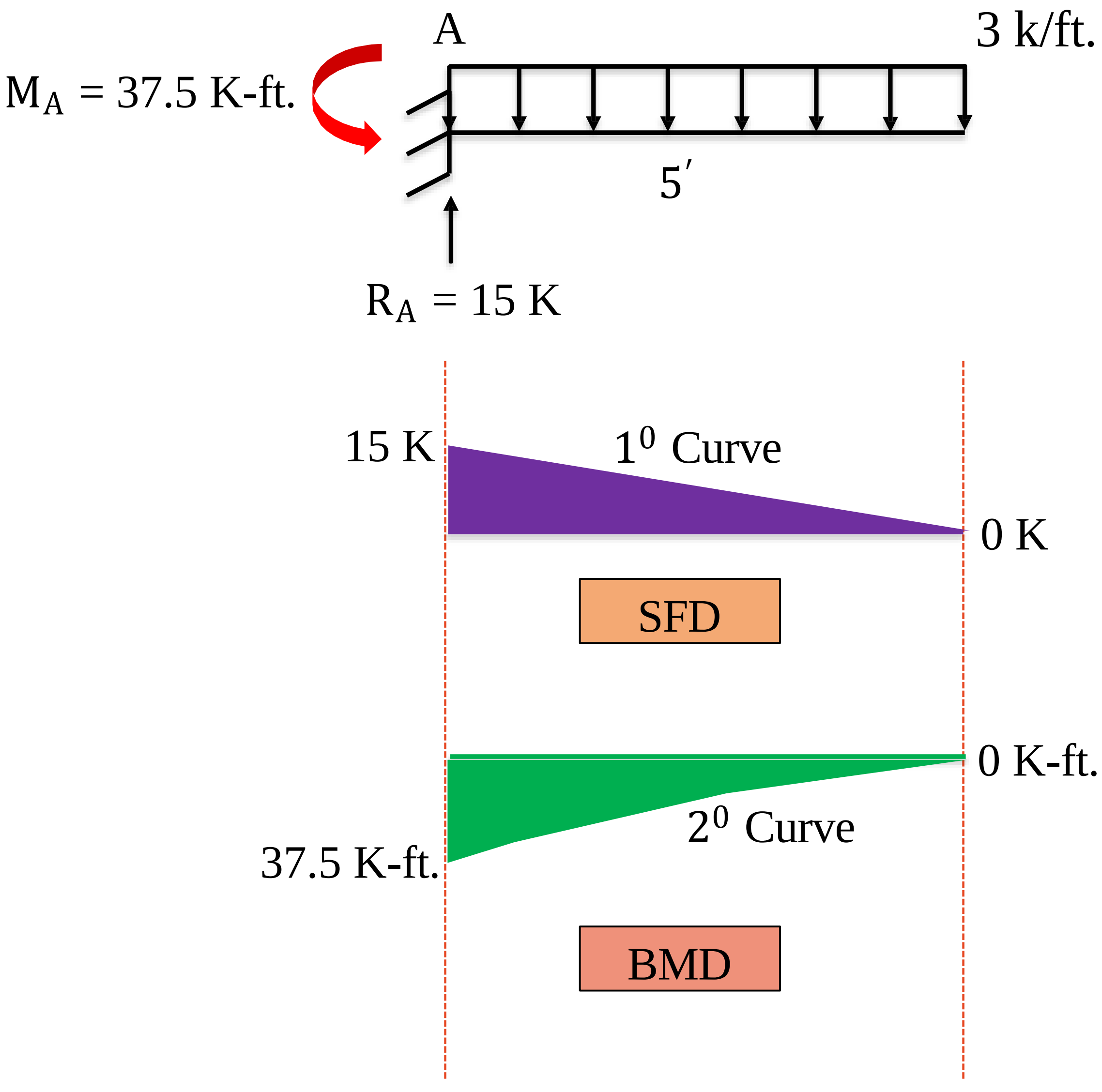


5 K
10′
5 K
5′
5′
3 k/ft.
5′
SFD
1⁰ Curve
BMD
SFD
1⁰ Curve
BMD
1⁰ Curve
SFD
2⁰ Curve
BMD

Problem-6: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$\sum F_y = 0$

$\Rightarrow 3 \text{ x } 5 - R_A = 0$

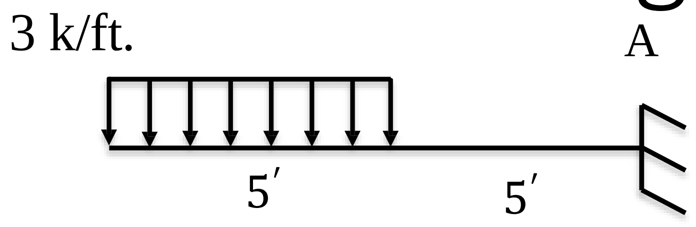
$\Rightarrow R_A = 15\text{ K.}$

$\sum M_A = 0$

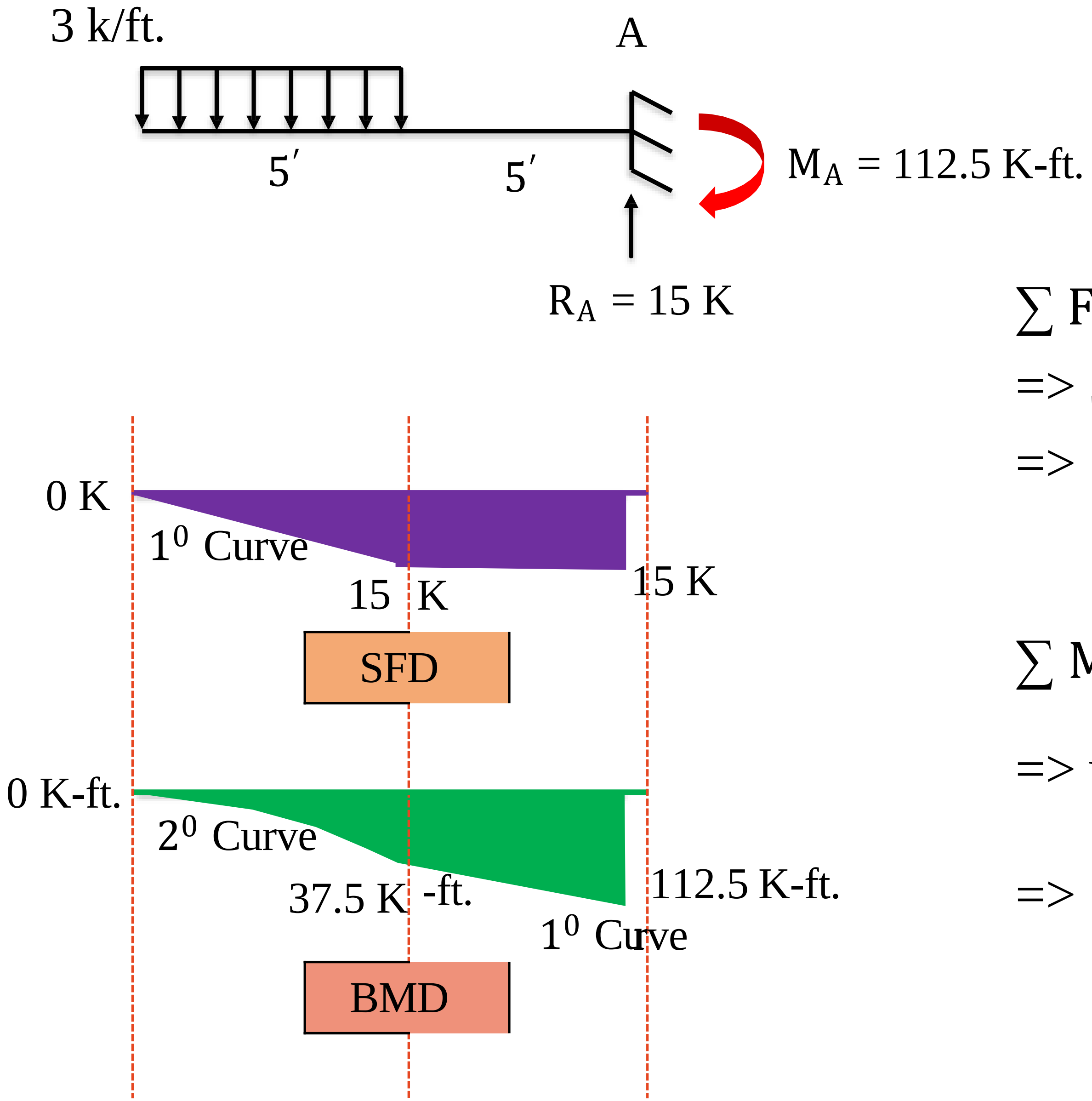
$\Rightarrow \frac{wl^2}{2} - M_A = 0$

$\Rightarrow M_A = \frac{3 \, x \, 5^2}{2} = 37.5 \text{ K-ft.}$

Problem-7: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$\sum F_y = 0$

$\Rightarrow 3 \text{ x } 5 - R_A = 0$

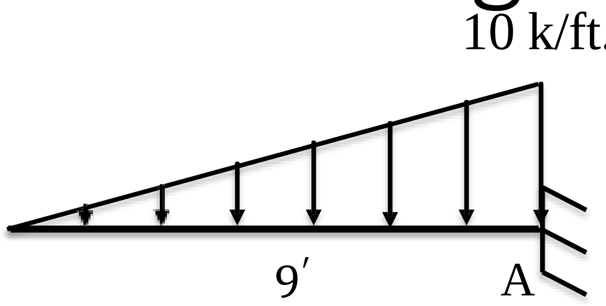
$\Rightarrow R_A = 15$ K.

$\sum M_A = 0$

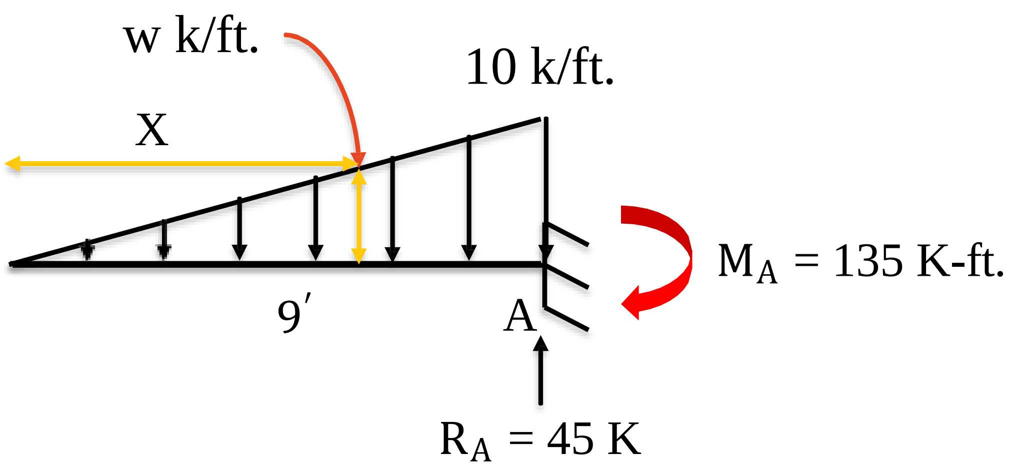
$\Rightarrow wl\,(5 + \frac{l}{2}) - M_A = 0$

$\Rightarrow M_A = 3 \text{ x } 5\,(5 + \frac{5}{2}) = 112.5$ K-ft.

Problem-8: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$$\sum F_y = 0$$

$$\Rightarrow 0.5 \times 9 \times 10 - R_A = 0$$

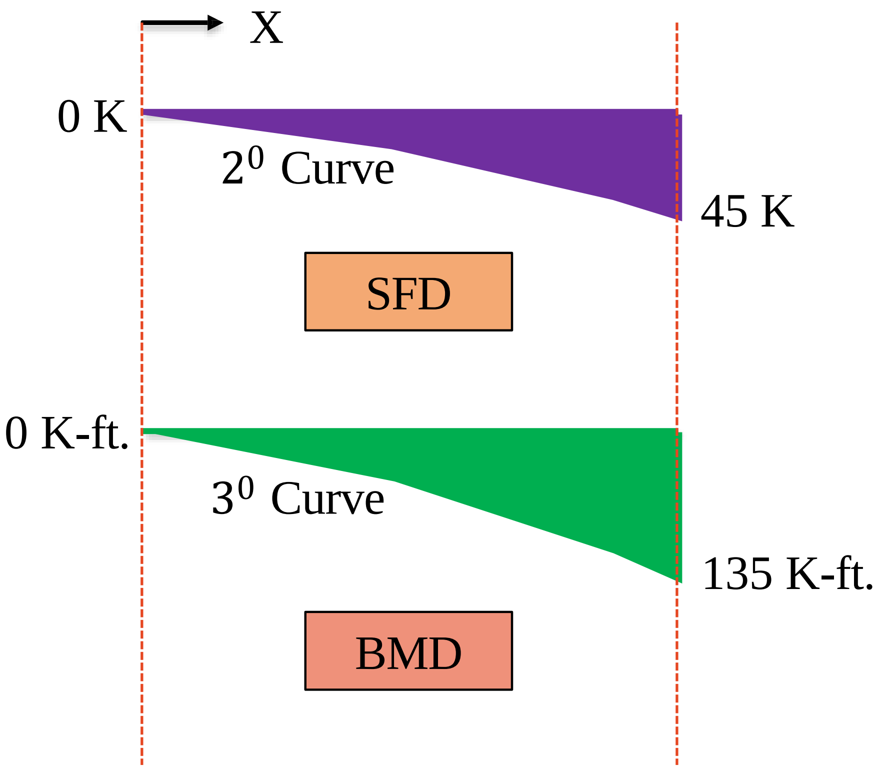
$$\Rightarrow R_A = 45 \text{ K.}$$

$$\sum M_A = 0$$

=> Area x cantorial distance - $M_A = 0$

$$\Rightarrow 0.5 \times 9 \times 10 \times \frac{9}{3} - M_A = 0$$

$$\Rightarrow M_A = 135 \text{ K-ft.}$$



$$V_x = -\frac{1}{2} \cdot x \cdot \left(\frac{10}{9} \cdot x\right)$$

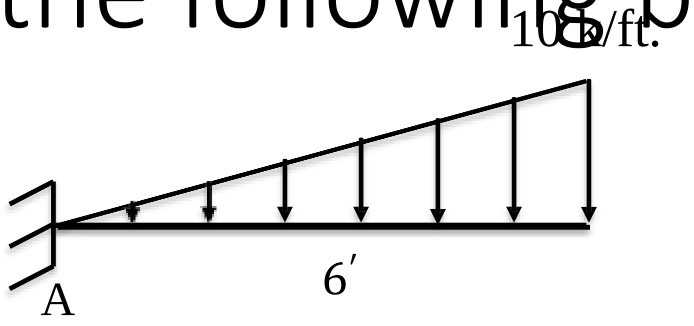
$$= -\frac{5x^2}{9} \quad [0 \le x \le 9]$$

$$M_x = -\frac{5x^2}{9} \cdot \frac{x}{3} \quad [0 \le x \le 9]$$

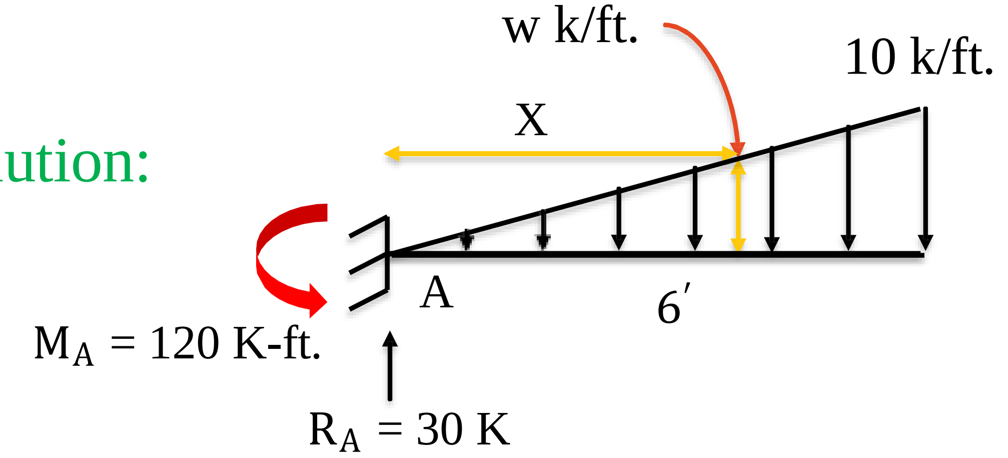
$$\frac{10}{9} = \frac{w}{x}$$

$$\Rightarrow w = \frac{10}{9} \cdot x$$

Problem-9: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$$V_x = 30 - \frac{1}{2} \cdot x \cdot (\frac{10}{6} \cdot x)$$

$$= 30 - \frac{5x^2}{6}$$

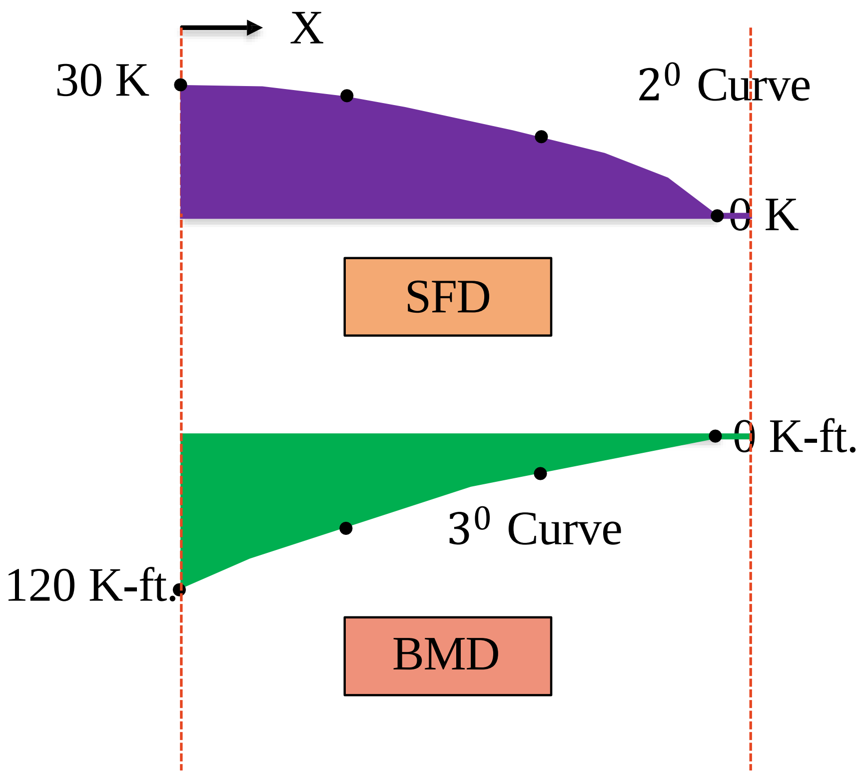
$$[0 \le x \le 6]$$

$$M_x = -120 + 30x - \frac{5x^2}{6} \cdot \frac{x}{3}$$

$$[0 \le x \le 6]$$

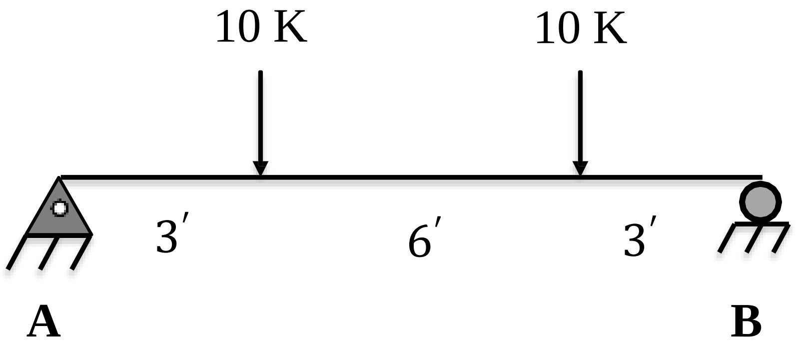
$$\frac{10}{6} = \frac{w}{x}$$

$$\Rightarrow w = \frac{10}{6} \cdot x$$

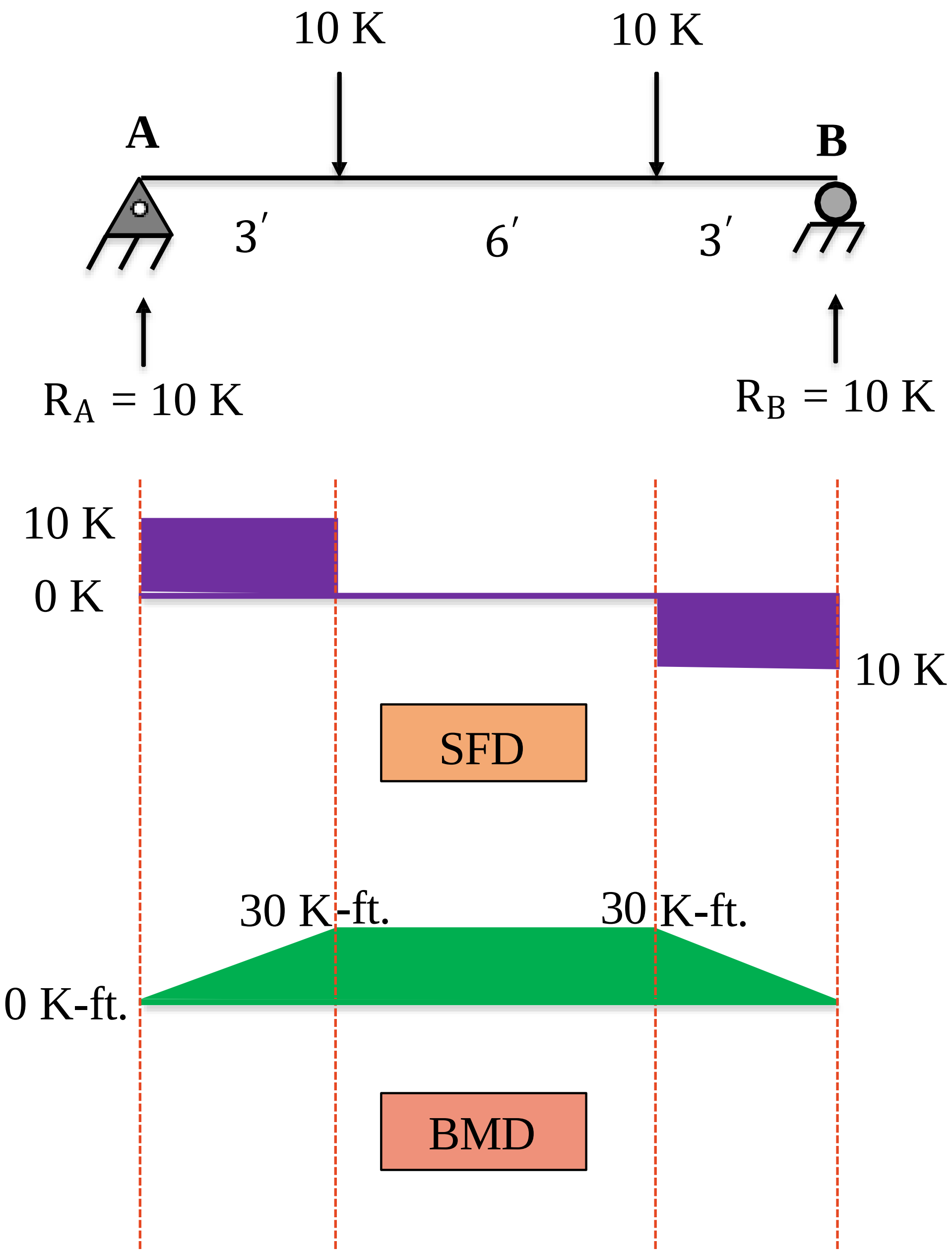


x	V_x (kip)	M_x (kip-ft.)
0	30	- 120
2	26.67	- 62.22
4	16.67	- 17.78
6	0	0

Problem-10: Find the SFD (Shear Force Diagram) & BMD (Bending Moment Diagram) of the following beam.



Solution:



$\sum M_A = 0$

$\Rightarrow 10 \times 3 + 10 \times (6 + 3) - R_B \times 12 = 0$

$\Rightarrow R_B \times 12 = 30 + 90 = 120$

$\Rightarrow R_B = 120/12 = 10$ K

$\sum F_y = 0$

$\Rightarrow R_A + R_B - 10 - 10 = 0$

$\Rightarrow R_A = 20 - R_B = 20 - 10 = 10$ K

Thin Walled Pressure Vessel

Week 8

Pages (61-73)


34
35

Introduction

- Pressure vessels are used to hold fluids such as liquids or gases that must be stored at relatively high pressures.
- Pressure vessels may be found in settings such as chemical plants, airplanes, power plants, submersible vehicles, and manufacturing processes.
- Boilers, gas storage tanks, pulp digesters, aircraft fuselages, water distribution towers,inflatable boats, distillation towers, expansion tanks, and pipelines are examples of pressure vessels.
- A vessel can be classified as thin walled if the ratio of the inside radius to the wall thickness is greater than about 10:1

Concept of Thick & Thin Cylinder or Shell

A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter.i. e., when the wall thickness, 't' is equal to or less than 'd/20', where 'd' is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

The following assumptions are made in order to derive the expressions for the stresses and strains in thin cylinders :

(i) The diameter of the cylinder is more than 20 times the thickness of the shell.

(ii) The stresses are uniformly distributed through the thickness of the wall.

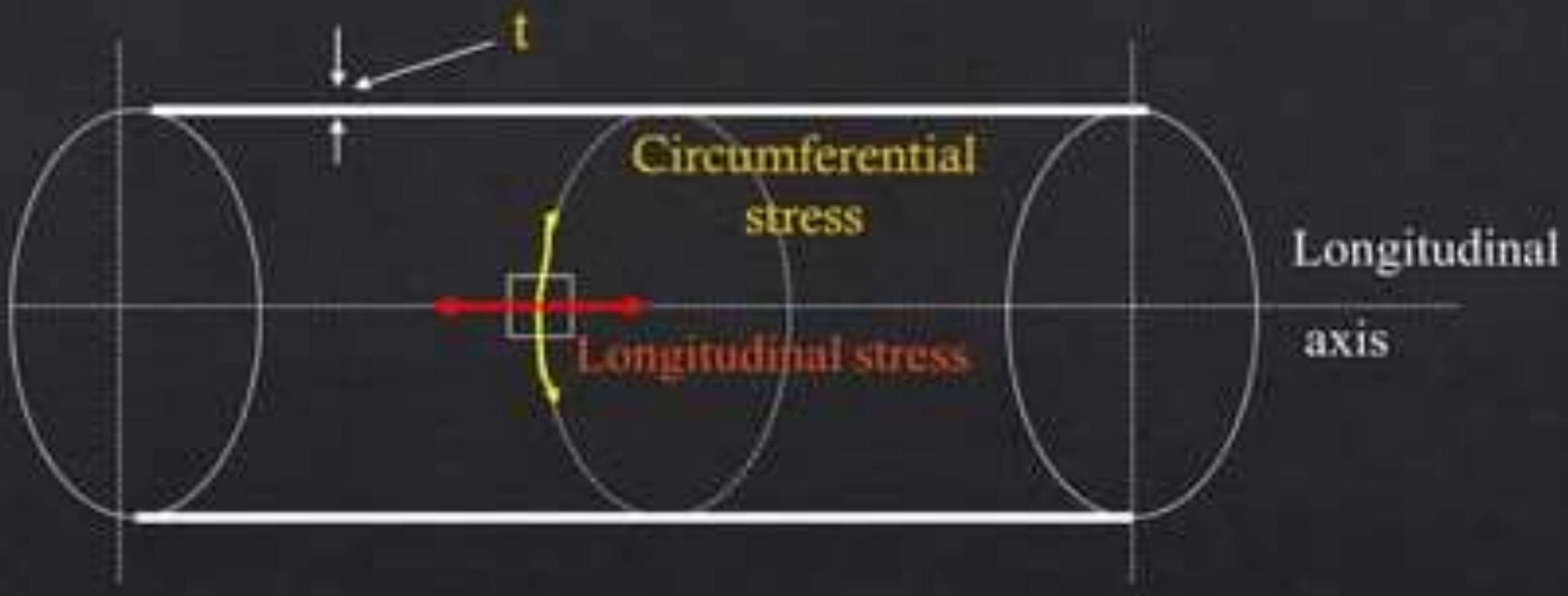
(iii) The ends of the cylindrical shell are not supported from sides.

These cylinders are subjected to fluid pressures. When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, three types of stresses are induced on three mutually perpendicular planes.

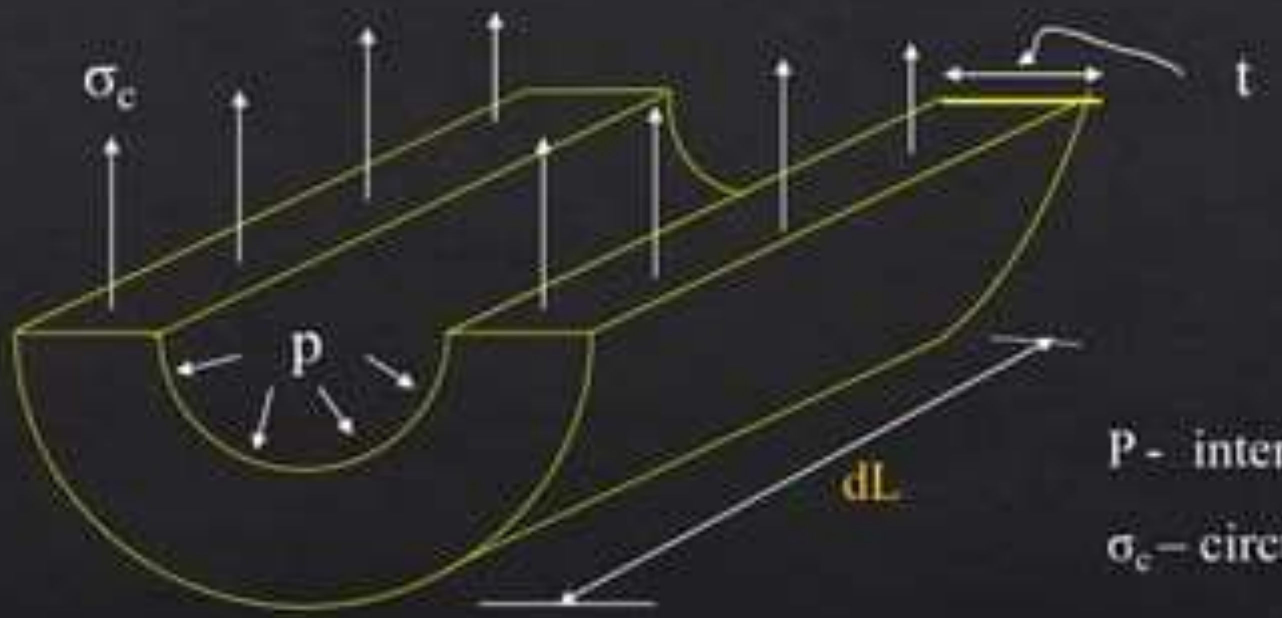
Hoop or Circumferential Stress (σ_C) – This is directed along the tangent to the circumference and tensile in nature. Thus, there will be increase in diameter.

Longitudinal Stress (σ_L) – This stress is directed along the length of the cylinder. This is also tensile in nature and tends to increase the length.

Radial Stress (σ_R) – It is compressive in nature. Its magnitude is equal to fluid pressure on the inside wall and zero on the outer wall if it is open to atmosphere.



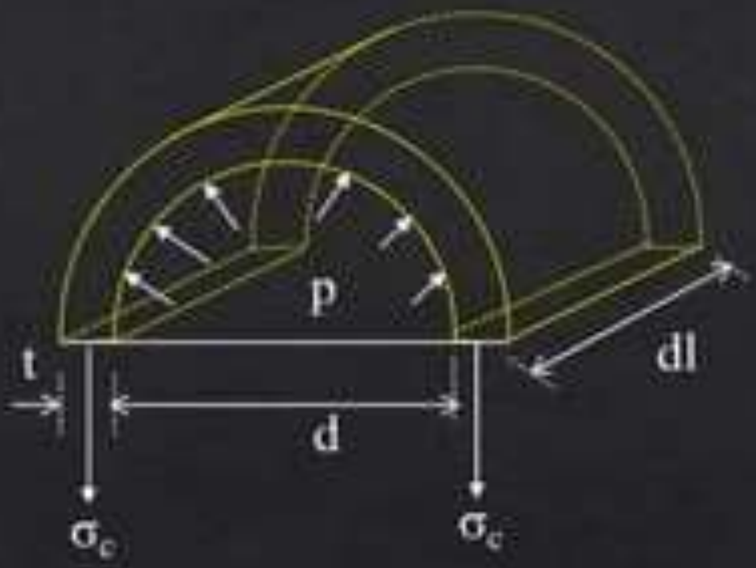
The stress acting along the circumference of the cylinder is called circumferential stresses whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stress



P - internal pressure (stress)

σ_c – circumferential stress

EVALUATION OF CIRCUMFERENTIAL or HOOP STRESS (σ_C):



Consider a thin cylinder closed at both ends and subjected to internal pressure 'p' as shown in the figure.

Let d=Internal diameter, t = Thickness of the wall

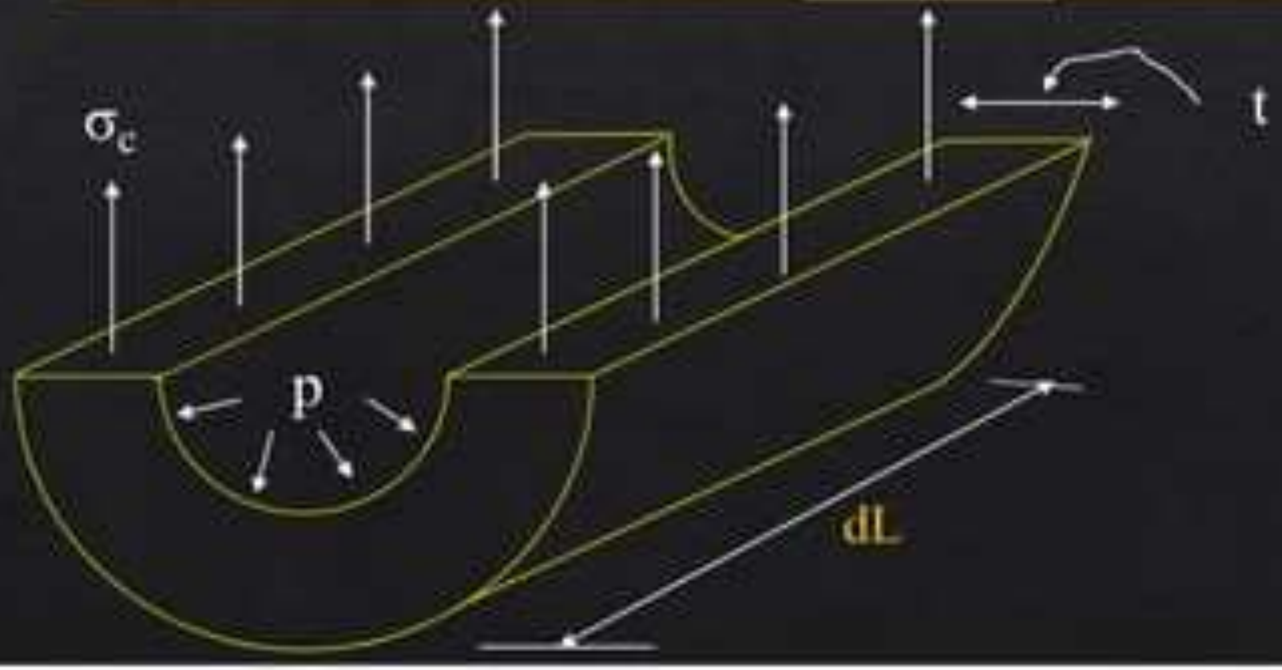
L = Length of the cylinder.

$\therefore$ Resisting force (due to circumferential stress σ_c) $= 2 \times \sigma_c \times t \times dl$

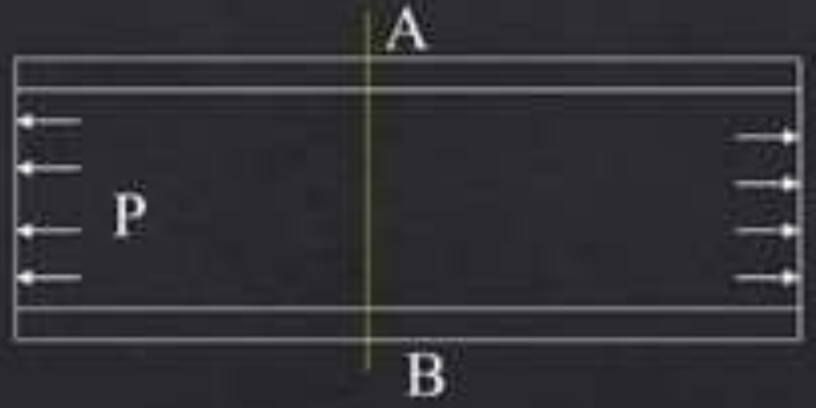
Under equilibrium, Resisting force = Bursting force

i.e., $2 \times \sigma_c \times t \times dl = p \times d \times dl$

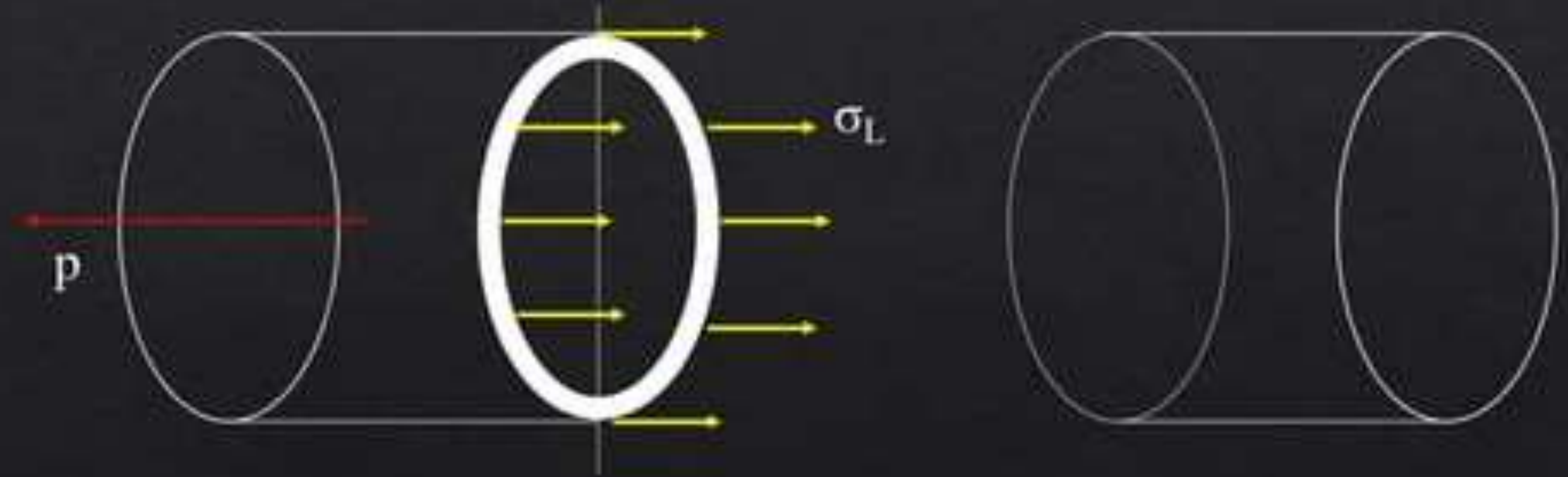
$$\therefore \text{Circumferential stress, } \sigma_c = \frac{p \times d}{2 \times t} \quad \text{.........................(1)}$$



LONGITUDINAL STRESS (σ_L):



The bursting of the cylinder takes place along the section AB



The force, due to pressure of the fluid, acting at the ends of the thin cylinder, tends to burst the cylinder as shown in figure

Under equillibrium, bursting force = resisting force

$$\text{i.e., } p \times \frac{\pi}{4} \times d^2 = \sigma_L \times \pi \times d \times t$$

$$\therefore \text{ Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \quad \text{..................(2)}$$

$$\text{From eqs (1) \& (2), } \quad \sigma_C = 2 \times \sigma_L$$

Problem 17.1. *A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine :*

(i) Longitudinal stress developed in the pipe, and

(ii) Circumferential stress developed in the pipe.

Sol. Given :

Dia. of pipe, $d = 1.5$ m

Thickness, $t = 1.5 \text{ cm} = 1.5 \times 10^{-2}$ m

Internal fluid pressure, $p = 1.2$ N/mm²

As the ratio $\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$, hence this is a case of thin cylinder.

Here unit of pressure (p) is in N/mm². Hence the unit of σ_1 and σ_2 will also be in N/mm².

(*i*) The longitudinal stress (σ_2) is given by equation (17.2) as,

$$\sigma_2 = \frac{p \times d}{4t}$$

$$= \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = 30 \text{ N/mm}^2. \quad \textbf{Ans.}$$

(*ii*) The circumferential stress (σ_1) is given by equation (17.1) as

$$\sigma_1 = \frac{pd}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}} = 60 \text{ N/mm}^2. \quad \textbf{Ans.}$$

Problem 17.2. *A cylinder of internal diameter 2.5 m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², determine the internal pressure of the gas.*

Sol. Given :

Internal dia. of cylinder, $d = 2.5$ m

Thickness of cylinder, $t = 5 \text{ cm} = 5 \times 10^{-2}$ m

Maximum permissible stress $= 80$ N/mm²

As maximum permissible stress is given. Hence this should be equal to circumferential stress (σ_1).

We know that the circumferential stress should not be greater than the maximum permissible stress. Hence take circumferential stress equal to maximum permissible stress.

$$\therefore \quad \sigma_1 = 80 \text{ N/mm}^2$$

Let p = Internal pressure of the gas

Using equation (17.1),

$$\sigma_1 = \frac{pd}{2t}$$

or

$$p = \frac{2t \times \sigma_1}{d} = \frac{2 \times 5 \times 10^{-2} \times 80}{2.5}$$

(Here unit of σ_1 is in N/mm², hence unit of p will also be in N/mm²)

$$= 3.2 \text{ N/mm}^2. \quad \textbf{Ans.}$$

Problem 17.5. *A water main 80 cm diameter contains water at a pressure head of 100 m. If the weight density of water is 9810 N/m³, find the thickness of the metal required for the water main. Given the permissible stress as 20 N/mm².* (AMIE, Summer 1974)

Sol. Given :

Dia. of main, $d = 80$ cm

Pressure head of water, $h = 100$ m

Weight density of water, $w = \rho \times g = 1{,}000 \times 9.81 = 9810\ \text{N/m}^3$

Permissible stress $= 20\ \text{N/mm}^2$

Permissible stress is equal to circumferential stress (σ_1)

or $\sigma_1 = 20\ \text{N/mm}^2$

Pressure of water inside the water main,

$$p = \rho \times g \times h = wh = 9810 \times 100\ \text{N/m}^2$$

Here σ_1 is in N/mm², hence pressure (p) should also be N/mm². The value of p in N/mm² is given as

$$p = \frac{9810 \times 100}{1000^2\ \text{mm}^2}\ \text{N/mm}^2 \qquad (\because\ 1\ \text{m} = 1000\ \text{mm})$$

$$= 0.981\ \text{N/mm}^2$$

Let t = Thickness of the metal required.

Using equation (17.1),

$$\sigma_1 = \frac{p \times d}{2 \times t} \qquad \text{(Here '}d\text{' is in cm hence '}t\text{' will also be in cm)}$$

$$\therefore \quad t = \frac{p \times d}{2 \times \sigma_1} = \frac{0.981 \times 80}{2 \times 20} = \mathbf{2\ cm.} \quad \textbf{Ans.}$$

Thermal Stress-Strain

Week 9

Pages (75-80)

- Temperature changes occur:

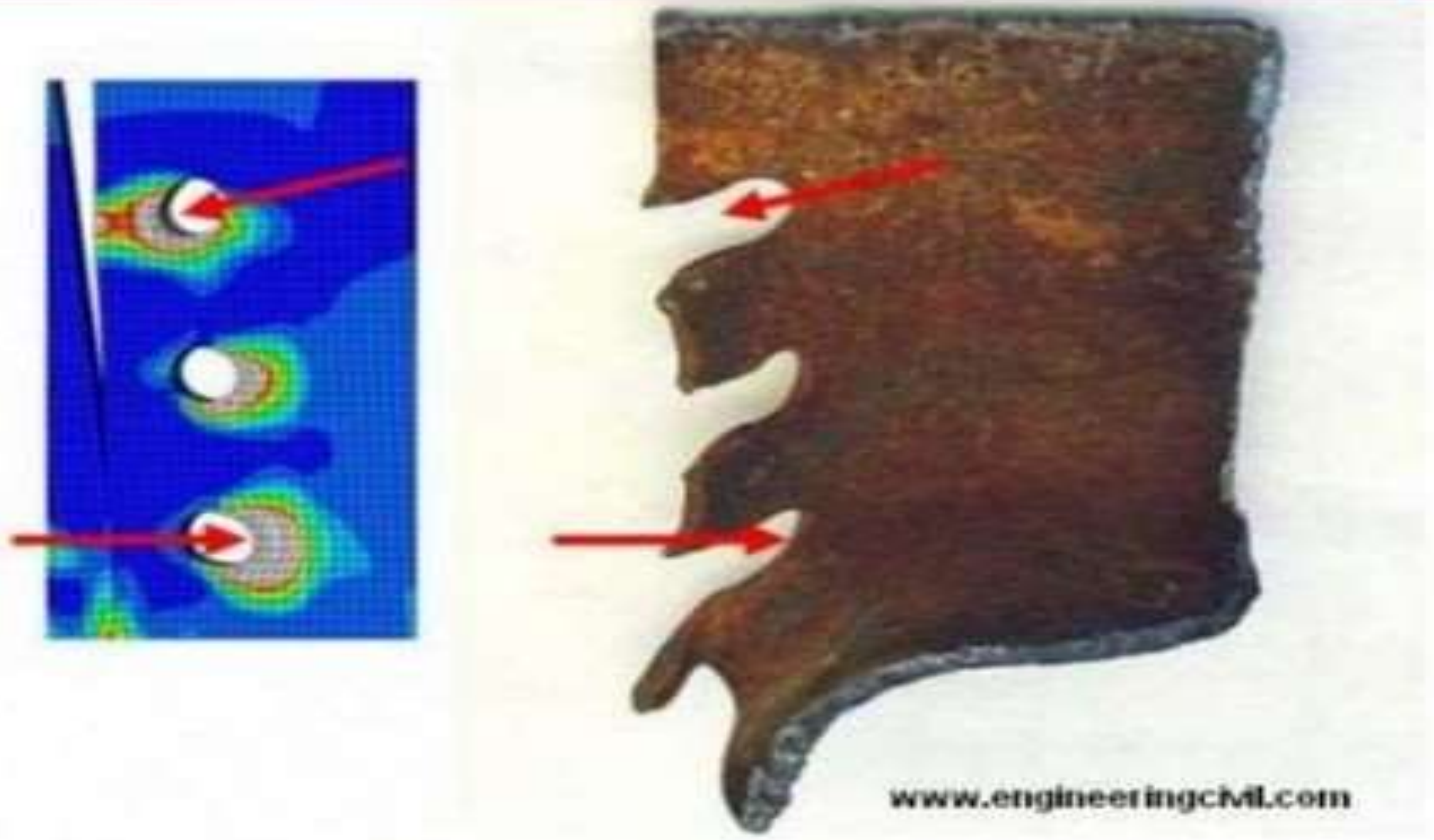


Figure 7. Comparison of Abaqus Model Results and Forensic Evidence.

Thermal effects

- Changes in temperature produce expansion or contraction of materials and result in *thermal strains* and *thermal stresses*
- For most structural materials, thermal strain ε_T is proportional to the temperature change ΔT :

$$\varepsilon_T = \alpha (\Delta T)$$

coefficient of thermal expansion

FIG. 2-19 Block of material subjected to an increase in temperature

- When a sign convention is needed for thermal strains, we usually assume that expansion is positive and contraction is negative

Thermal Stress

- Suppose we have a bar subjected to an axial load. We will then have:

$$\varepsilon = \sigma / E$$

- Also suppose that we have an identical bar subjected to a temperature change ΔT.

We will then have:

$$\varepsilon_T = \alpha (\Delta T)$$

- Equating the above two strains we will get:

$$\sigma = E \alpha (\Delta T)$$

- We now have a relation between axial stress and change in temperature

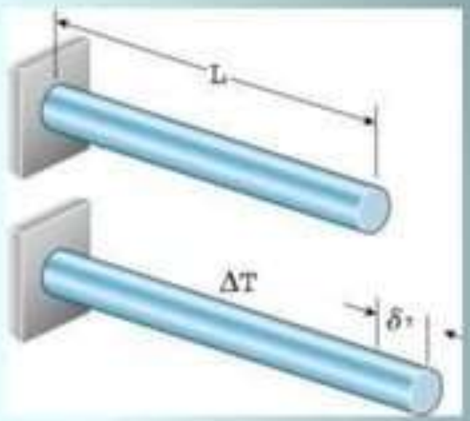


FIG. 2-20 Increase in length of a prismatic bar
due to a uniform increase
in temperature (Eq. 2-16)

- Assume that the material is homogeneous and isotropic and that the temperature increase ΔT is uniform throughout the block.
- We can calculate the increase in *any* dimension of the block by multiplying the original dimension by the thermal strain.

FIG. 2-19 Block of material subjected to an increase in temperature

$$\delta_T = \varepsilon_T L = \alpha (\Delta T) L$$

Temperature – Displacement relation

Thermal Strain

As in the case of lateral strains, thermal strains do not induce stresses unless they are constrained. The total strain in a body experiencing thermal stress may be divided into two components:

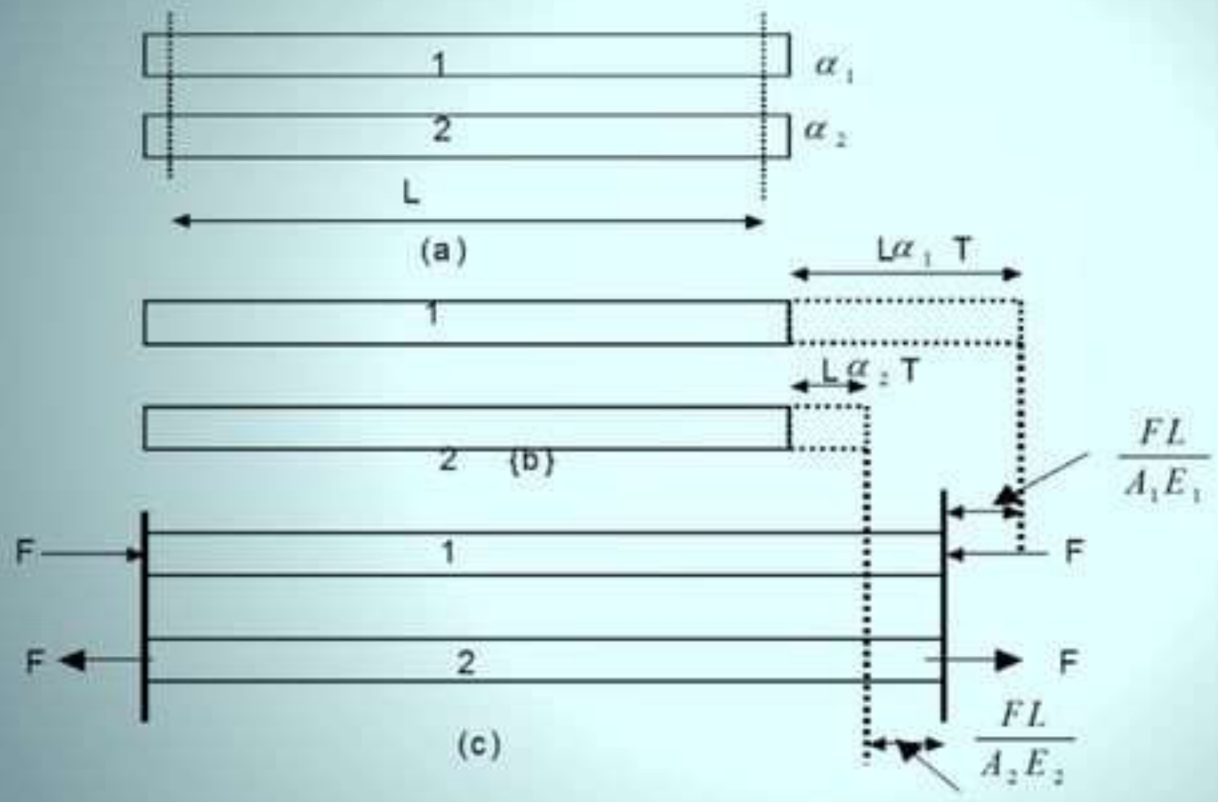
Strain due to stress, ε_σ and

That due to temperature, ε_T .

Thus: $\varepsilon = \varepsilon_\sigma + \varepsilon_T$

$$\varepsilon = \frac{\sigma}{E} + \alpha T$$

Temperature stresses in composite bars



Expression for bending Stress

Week 10

Pages (82-94)

Expression for Bending Stress

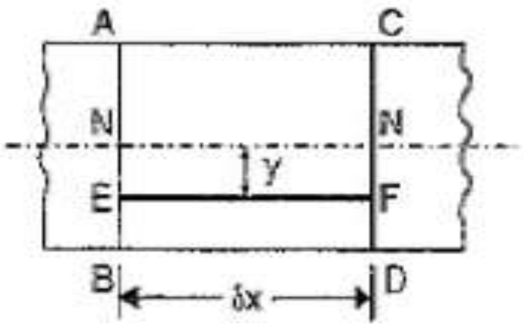


Fig. 3

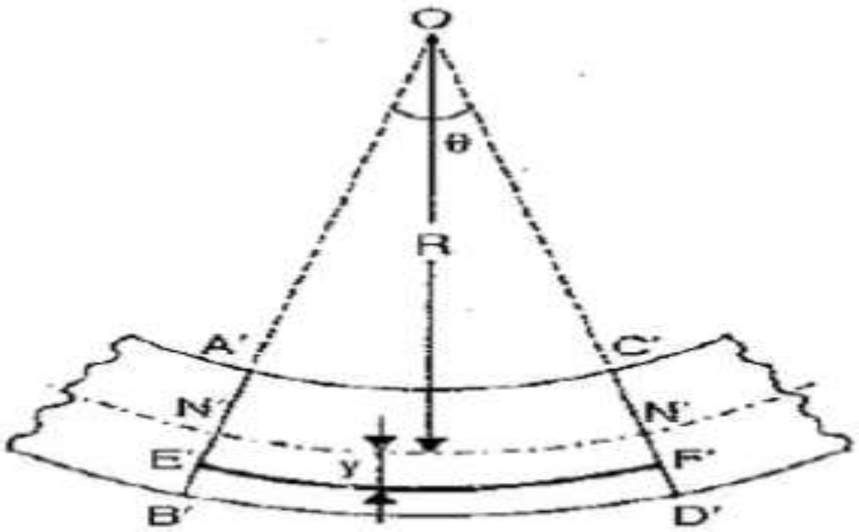
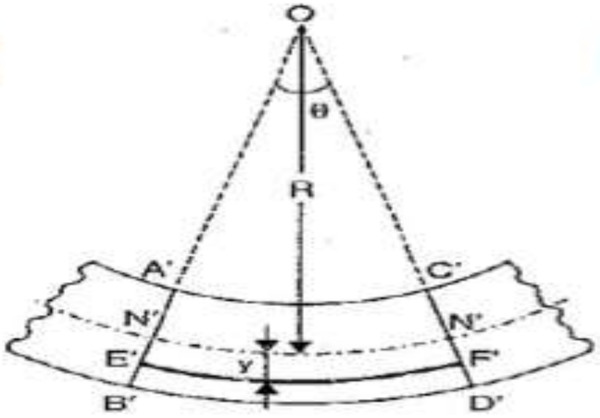


Fig. 4

Expression for Bending Stress

- Consider a small length δx of a beam subjected to simple bending.
- Due to the action of bending, the part of length δx will be deformed as shown in Figure 4.
- Let, $A^{/} B^{/}$ and $C^{/} D^{/}$ meet at O.
- R = radius of neutral layer $N^{/}N^{/}$
- θ = angle subtended at O by $A^{/}B^{/}$ and $C^{/}D^{/}$.
- Consider a layer EF at a distance y below the neutral layer NN. After bending this layer will be elongated to $E^{/}F^{/}$.
- Original length of layer EF = δx
- Length of neutral layer NN = δx.
- After bending length of neutral layer remains the same.

Expression for Bending Stress



- Therefore, $NN = N^{/}N^{/} = \delta x$.
- Now from Figure 4, $N^{/}N^{/} = R^{*}\ \theta$
- and $E^{/}F^{/} = (R+y)^{*}\ \theta$
- but $NN = N^{/}N^{/} = \delta x$
- Hence, $\delta x = R^{*}\ \theta$
- Increase in the length of the layer $EF = E^{/}F^{/} - EF$
- $= (R+y)^{*}\ \theta - R^{*}\ \theta = y^{*}\ \theta$

$$Strain\ in\ the\ layer\ EF = \frac{Increase\ in\ length}{Original\ length} = \frac{y\theta}{R\theta} = \frac{y}{R}$$

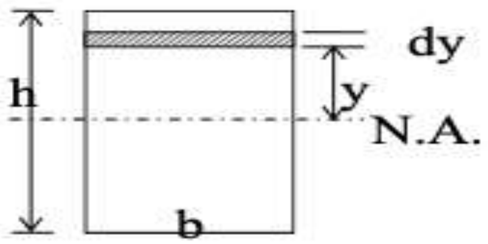
- Since R is constant, strain in a layer is proportional to the distance from the neutral axis.

Expression for Bending Stress

- **Stress Variation**
- σ = Stress in the layer
- E = Modulus of elasticity
- Then, stress = Strain x modulus of elasticity
- $$\sigma = \frac{E}{R}y \qquad \therefore \frac{\sigma}{y} = \frac{E}{R}$$
- Since E and R are constants, therefore stress in any layer is directly proportional to the distance of the layer from the neutral surface.

Expression for Bending Stress

- **Neutral Axis and Moment of Resistance**



- Figure shows the cross section of a beam. Consider a small layer of area ***dA*** at a distance y from the neutral axis.
- Now force on the layer = stress in the layer * area of the layer =

$$\sigma * dA = \frac{E}{R} y * dA$$

- Total force on the beam section = $\int \frac{E}{R} y * dA = \frac{E}{R} \int y * dA$

Expression for Bending Stress

- **Moment of Resistance**
- Due to pure bending layers above the N.A. are subjected to compressive stress whereas the layers below the N.A. are subjected to tensile stresses.
- Due to these stresses, the forces will be acting on the layers .
- These forces will have a moment about the neutral axis. The total moment of these forces about the N.A. for a section is known as the moment of resistance of that section.
- Moment of the force of the layer about N.A.
- $= \frac{E}{R} y * dA * y \quad = \frac{E}{R} y^2 dA$

Expression for Bending Stress

- Total moment of the forces on the section of the beam =

$$\int \frac{E}{R} y^2 dA$$

- Let M = external moment applied on the beam.
- For equilibrium, external moment = internal moment

$$M = \frac{E}{R} \int y^2 dA$$

- The moment of inertia of the area $I = \int y^2 dA$

- $\therefore M - \frac{EI}{R} \text{ or } \frac{M}{I} - \frac{E}{R}$

Expression for Bending Stress

$$\therefore \frac{M}{I} = \frac{\sigma}{y} \qquad since \left[\frac{\sigma}{y} = \frac{E}{R}\right]$$

$$Bending\ stress\ \sigma = \frac{My}{I}$$

Important Points of the Flexure Formula

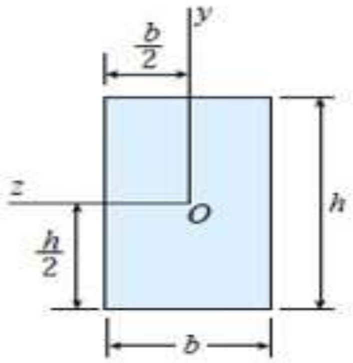
- The cross section of a straight beam *remains plane* when the beam deforms due to bending. This causes tensile stress on one portion of the cross section and compressive stress on the other portion. In between these portions, there exists the *neutral axis* which is subjected to *zero stress*.
- Due to the deformation, the *longitudinal strain* varies *linearly* from zero at the neutral axis to a maximum at the outer fibers of the beam. Provided the material is homogeneous and linear elastic, then the *stress* also varies in a *linear* fashion over the cross section.

Important Points of the Flexure Formula

- The neutral axis passes through the *centroid* of the cross-sectional area. This result is based on the fact that the resultant normal force acting on the cross section must be zero.
- The flexure formula is based on the requirement that the resultant internal moment on the cross section is equal to the moment produced by the normal stress distribution about the neutral axis.

Section Modulus

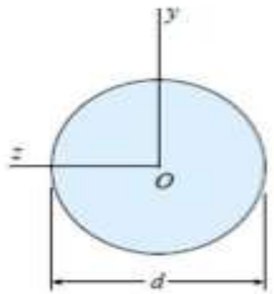
- The term I/c in expression for maximum bending stress is known as section modulus.
- The advantage of expressing the maximum stresses in terms of section moduli arises from the fact that each section modulus combines the beam's relevant cross-sectional properties into a single quantity.



$I = bh^3/12$ $c = h/2$

Section modulus, $z = I/c = bh^2/6$

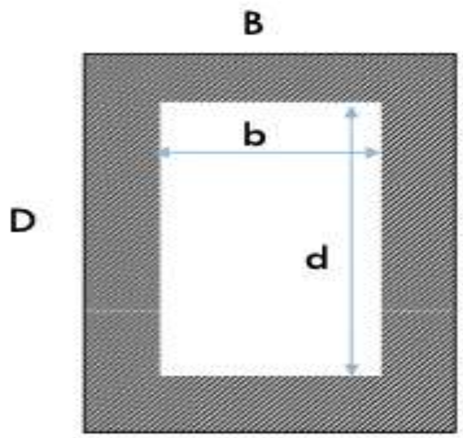
Section Modulus



$I = \pi d^4/64 \qquad c = d/2$

Section modulus, $z = I/c = \pi d^3/32$

Section Modulus



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

c = D/2

Section modulus Z = I/c

$$Z = \frac{1}{6D}\left(BD^3 - bd^3\right)$$

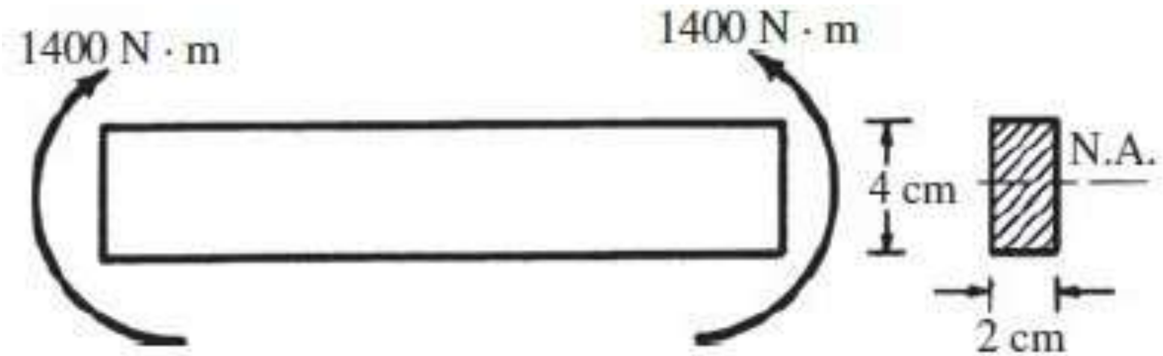
Bending stress related problem

Week 11

Pages (96-102)

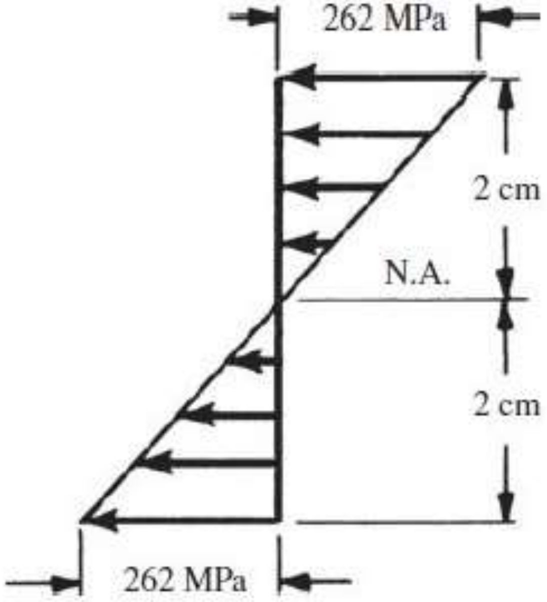
Example #1

- A beam is loaded by a couple of 1400 N-m at each of its ends, as shown in Figure. Determine the maximum bending stress in the beam and indicate the variation of bending stress over the depth of the beam.



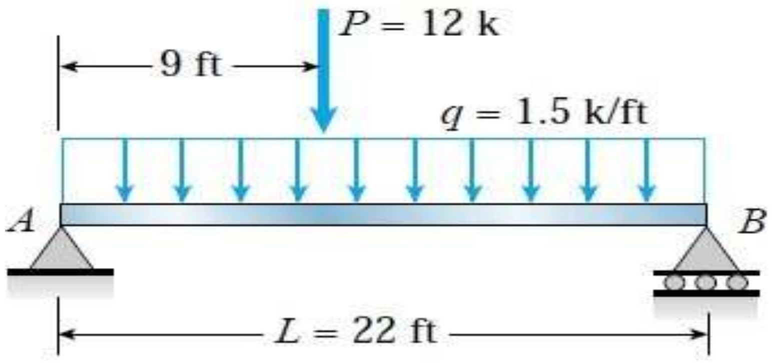
- S

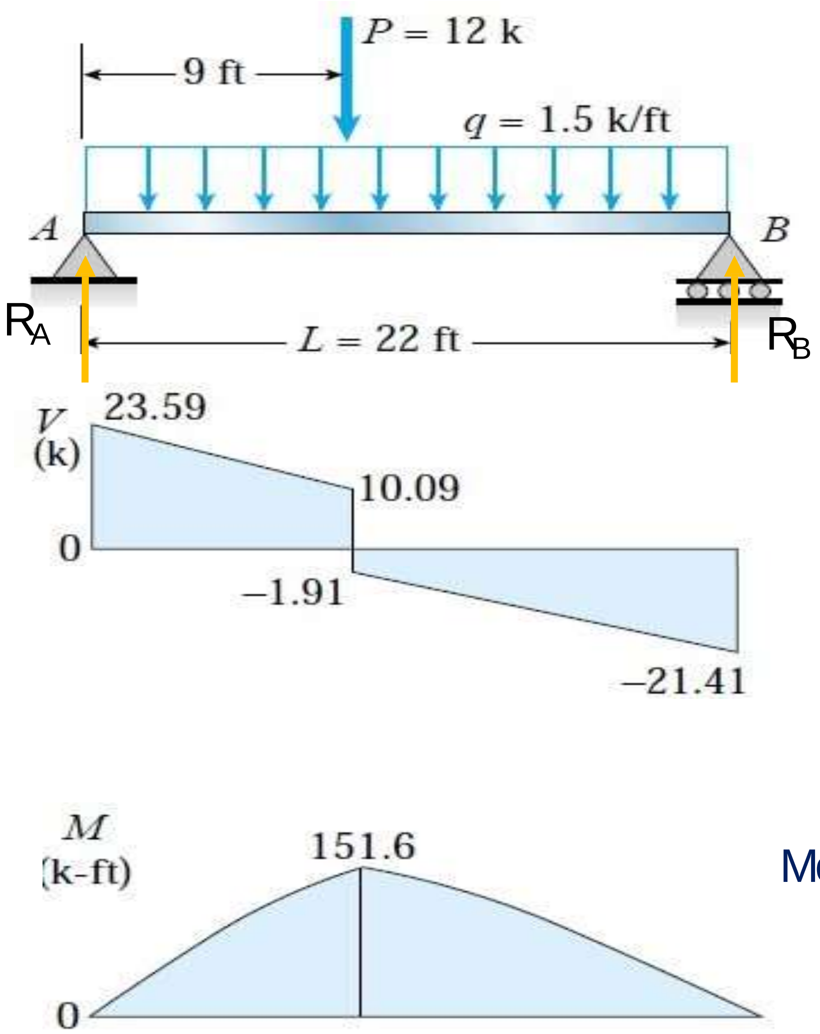
$$\sigma = \frac{Mc}{I} = \frac{1400 \times 0.02}{0.02 \times 0.04^3/12} = 262 \times 10^6 \text{ Pa}$$



Example #2

- A simple beam *AB of span length 22 ft supports a uniform load* of intensity *1.5 k/ft and a concentrated load 12 k*. The concentrated load acts at a point 9.0 ft from the left-hand end of the beam. The beam has a cross section of width *b = 9.0 in. and height h = 27 in.*





$\sum M_A = 0$

$1.5 * 22 * 22/2 + 12 * 9 - R_B * 22 = 0$

$R_B = 21.41k$ $R_A = 23.59k$

Shear at point load = $R_A - 1.5*9$

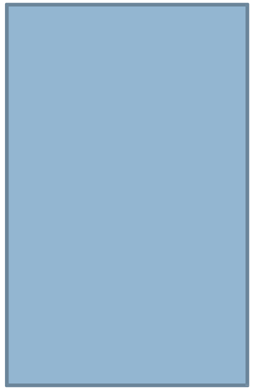
= $23.59 - 13.5$

= 10.09 k

Moment at point load = $R_A * 9 - 1.5*9*9/2$

= $23.59*9 - 13.5*4.5$

= 151.6 k-ft



h = 27 in.

b = 9 in.

Moment of inertia $I = bh^3/12$

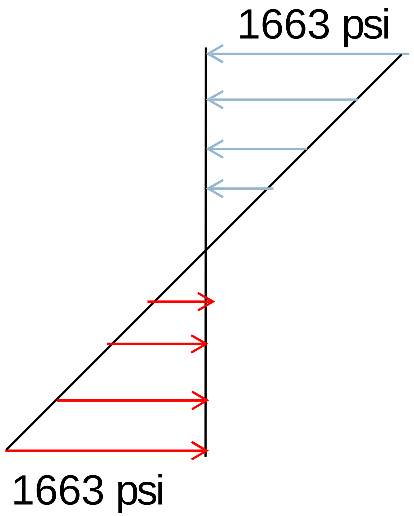
$= 9*(27)^3/12$

$= 14{,}762.25$ in^4

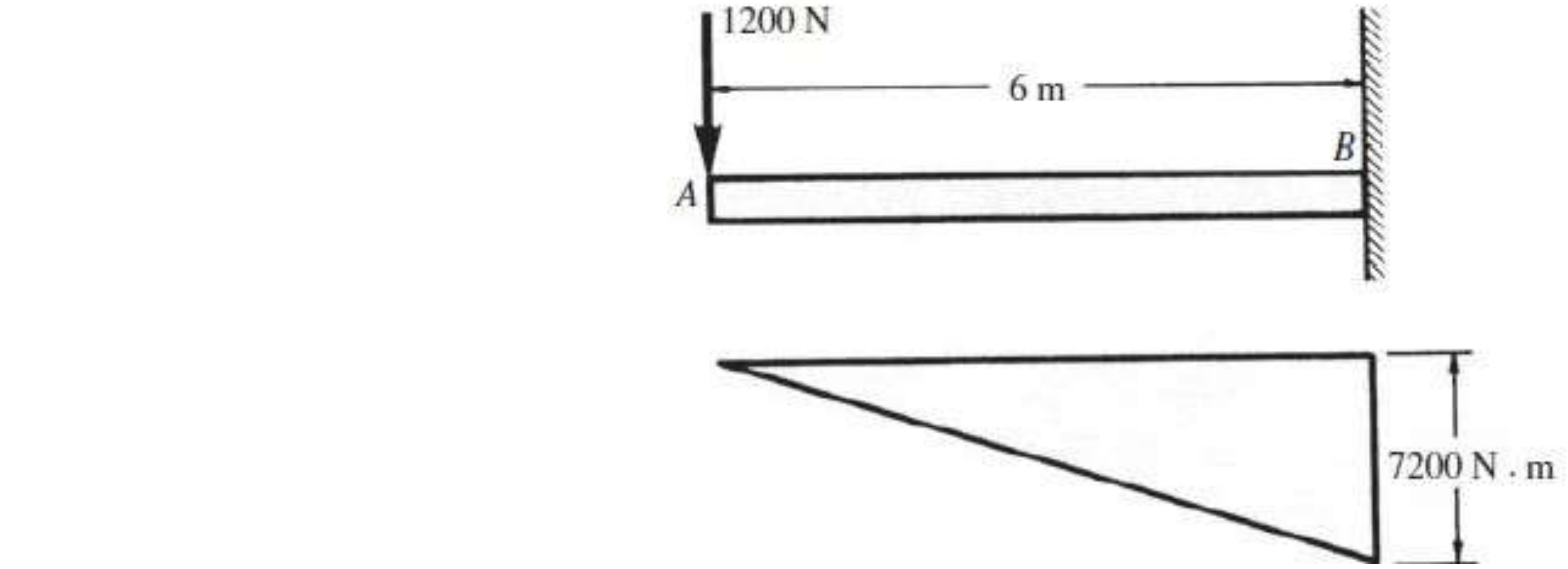
$c = h/2 = 13.5$ in

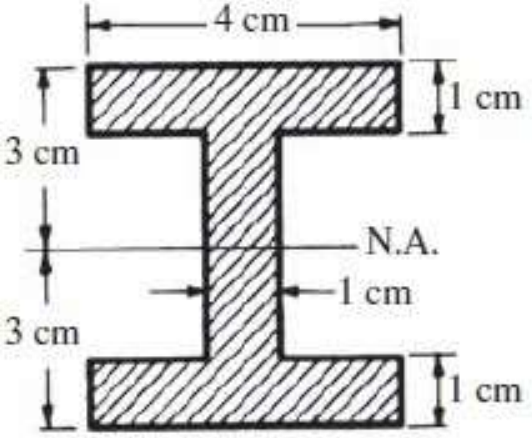
Maximum bending stress

$$\sigma = \frac{Mc}{I} = \frac{151.6*12*13.5}{14762.25} = 1.663 ksi$$



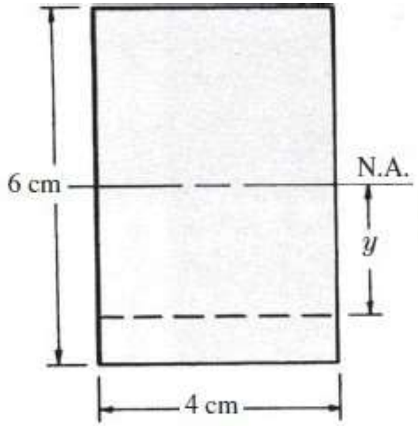
A steel cantilever beam 6 m in length is subjected to a concentrated load of 1200 N acting at the free end of the bar. Determine the magnitude and location of the maximum tensile and compressive bending stresses in the beam.





$$I = \frac{4 \times 6^3}{12} - \frac{3 \times 4^3}{12} = 56\ \text{cm}^4$$

$$\sigma = \frac{Mc}{I} = \frac{7200 \times 0.03}{56 \times 10^{-8}} = 386 \times 10^6\ \text{Pa}$$



$$\sigma = \frac{My}{I} = \frac{7200 \times 0.03}{0.04 \times 0.06^3/12} = 300 \times 10^6\ \text{Pa}$$

Torsion

Week 12-13

Pages (104-108)

Torsion

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.

TORSIONAL SHEARING STRESS, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is

$$\tau = \frac{T\rho}{J} \text{ and } \tau_{max} = \frac{Tr}{J}$$

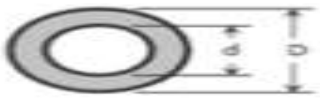
where J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft:

$$J = \frac{\pi}{32}D^4$$
$$\tau_{max} = \frac{16T}{\pi D^3}$$

For hollow cylindrical shaft:

$$J = \frac{\pi}{32}(D^4 - d^4)$$
$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$



ANGLE OF TWIST

The angle θ through which the bar length L will twist is

$$\theta = \frac{TL}{JG} \text{ in radians}$$

where T is the torque in N·mm, L is the length of shaft in mm, G is shear modulus in MPa, J is the polar moment of inertia in mm^4, D and d are diameter in mm, and r is the radius in mm.

POWER TRANSMITTED BY THE SHAFT

A shaft rotating with a constant angular velocity ω (in radians per second) is being acted by a twisting moment T. The power transmitted by the shaft is

$$P = T\omega = 2\pi T f$$

where T is the torque in N·m, f is the number of revolutions per second, and P is the power in watts.

Solved Problems in Torsion

Problem 304

A steel shaft 3 ft long that has a diameter of 4 in. is subjected to a torque of 15 kip·ft. Determine the maximum shearing stress and the angle of twist. Use $G = 12 \times 10^6$ psi.

Solution 304

$$\tau_{max} = \frac{16T}{\pi D^3} = \frac{16(15)(1000)(12)}{\pi(4^3)}$$

$$\tau_{max} = 14\,324 \text{ psi}$$

$$\tau_{max} = \mathbf{14.3 \text{ ksi}}$$

$$\theta = \frac{TL}{JG} = \frac{15(3)(1000)(12^2)}{\frac{1}{32}\pi(4^4)(12\times10^6)}$$

$$\theta = 0.0215 \text{ rad}$$

$$\theta = \mathbf{1.23^\circ}$$

Problem 305

What is the minimum diameter of a solid steel shaft that will not twist through more than 3° in a 6-m length when subjected to a torque of 12 kN·m? What maximum shearing stress is developed? Use G = 83 GPa.

Solution 305

$$\theta = \frac{TL}{JG}$$

$$3^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{12(6)(1000^3)}{\frac{1}{32}\pi d^4(83000)}$$

$$d = \mathbf{113.98\ mm}$$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(12)(1000^2)}{\pi(113.98^3)}$$

$$\tau_{max} = \mathbf{41.27\ MPa}$$

Problem 306

A steel marine propeller shaft 14 in. in diameter and 18 ft long is used to transmit 5000 hp at 189 rpm. If $G = 12 \times 10^6$ psi, determine the maximum shearing stress.

Solution 306

$$T = \frac{P}{2\pi f} = \frac{5000(396\,000)}{2\pi(189)}$$

$$T = 1\,667\,337.5\ \text{lb·in}$$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(1\,667\,337.5)}{\pi(14^3)}$$

$$\tau_{max} = \mathbf{3094.6\ psi}$$

Problem 307

A solid steel shaft 5 m long is stressed at 80 MPa when twisted through 4°. Using G = 83 GPa, compute the shaft diameter. What power can be transmitted by the shaft at 20 Hz?

Solution 307

$$\theta = \frac{TL}{JG}$$

$$4^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{T(5)(1000)}{\frac{1}{32}\pi d^4(83\,000)}$$

$$T = 0.1138d^4$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$80 = \frac{16(0.1138d^4)}{\pi d^3}$$

$$d = 138\text{ mm}$$

$$T = \frac{P}{2\pi f}$$

$$0.1138d^4 = \frac{P}{2\pi(20)}$$

$$P = 14.3d^4 = 14.3(138^4)$$

$$P = 5\,186\,237\,285\text{ N-mm/sec}$$

$$P = 5\,186\,237.28\text{ W}$$

$$P = 5.19\text{ MW}$$

Problem 308

A 2-in-diameter steel shaft rotates at 240 rpm. If the shearing stress is limited to 12 ksi, determine the maximum horsepower that can be transmitted.

Solution 308

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$12(1000) = \frac{16T}{\pi(2^3)}$$

$$T = 18\,849.56\text{ lb-in}$$

$$T = \frac{P}{2\pi f}$$

$$18\,849.56 = \frac{P(396\,000)}{2\pi(240)}$$

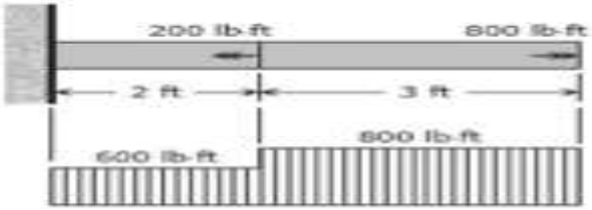
$$P = 71.78\text{ hp}$$

Problem 318

A solid aluminum shaft 2 in. in diameter is subjected to two torques as shown in Fig. P-318. Determine the maximum shearing stress in each segment and the angle of rotation of the free end. Use $G = 4 \times 10^6$ psi.

Figure P-318

Solution 318



$$\tau_{max} = \frac{16T}{\pi D^3}$$

For 2-ft segment:

$$\tau_{max2} = \frac{16(600)(12)}{\pi(2^3)} = 4583.66 \text{ psi}$$

For 3-ft segment:

$$\tau_{max3} = \frac{16(800)(12)}{\pi(2^3)} = 6111.55 \text{ psi}$$

$$\theta = \frac{TL}{JG}$$

$$\theta = \frac{1}{JG}\sum TL$$

$$\theta = \frac{1}{\frac{1}{32}\pi(2^4)(4\times10^6)}[600(2) + 800(3)]\,(12^2)$$

$$\theta = 0.0825 \text{ rad}$$

$$\theta = 4.73°$$

Mohr Circle construction Principle

Week 14

Pages (110-112)

MOHR'S Circle For Plane Stress

The transformation equations for plane stress can be represented in graphical form by a plot known as **Mohr's circle**. It is so named in honor of the German Professor in civil engineering Otto Christian Mohr (1835-1918), who in 1895 suggested its use in the stress analysis.

In this section, we will show how to apply the equations for plane-stress transformation using a graphical procedure that is often convenient to use and easy to remember.

If we write the earlier mentioned equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

in the form:

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

then the parameter θ can be eliminated by squaring each equation and adding them together. The result is:

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Since σx, σy, Txy are known constants, then the above equation can be written in a more compact form as:

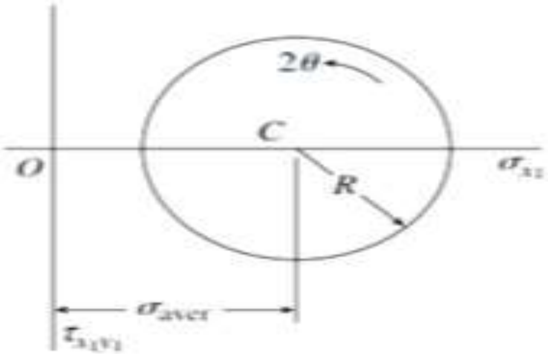
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

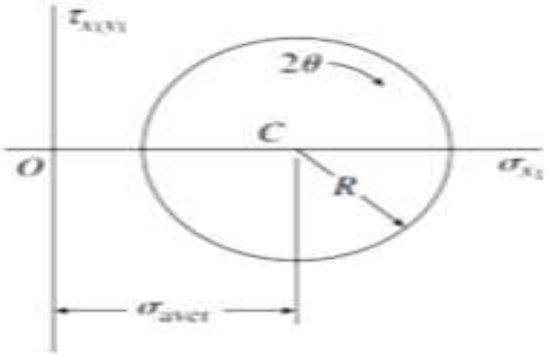
$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2 \quad \text{or} \quad (\sigma_{x_1} - \sigma_{aver})^2 + \tau_{x_1y_1}^2 = R^2 \Longrightarrow R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

If we establish coordinate axes, σ positive to the right and T positive downward, and then plot the above equation, it will be seen that this equation represents a circle having a radius R and center on the s axis at point C (σ avg, 0)

Mohr's circle can be plotted from in either of two forms. In the first form of Mohr's circle, we plot the normal stress σ_{x1} positive to the right and the shear stress τ_{x1y1} positive downward. The advantage of plotting shear stresses positive downward is that the angle 2θ on Mohr's circle will be positive when counterclockwise, which agrees with the positive direction of 2θ in the derivation of the transformation equations.

In the second form of Mohr's circle, τ_{x1y1} is plotted positive upward but the angle 2θ is now positive clockwise which is opposite to its usual positive direction. Both forms of Mohr's circle are mathematically correct, and either one can be used. However, it is easier to visualize the orientation of the stress element if the positive direction of the angle 2θ is the same in Mohr's circle as it is for the element itself. Furthermore, a counterclockwise rotation agrees with the customary right-hand rule for rotation. Therefore, we will choose the first form of Mohr's circle in which positive shear stress is plotted downward and a positive angle 2θ is plotted counterclockwise.



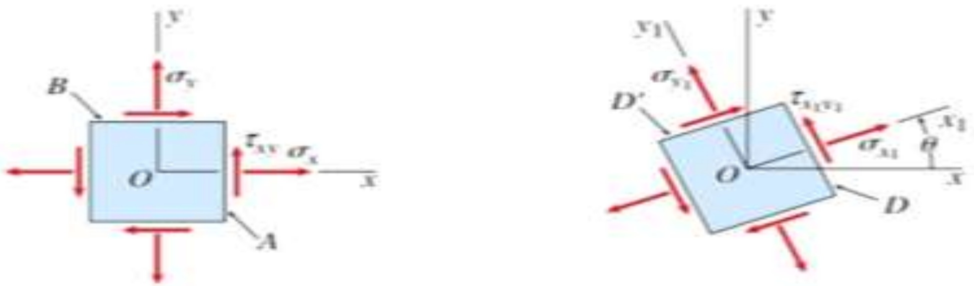


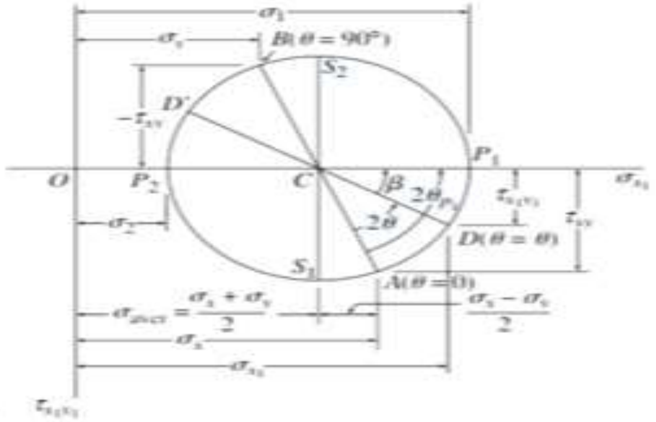
Construction of Mohr's circle of stress:

With σx, σy, and Txy known, the **procedure for constructing Mohr's circle** is as follows:

1. Draw a set of coordinate axes with σx1 (positive to the right) and Tx1y1 as ordinate (positive downward).
2. Locate the center C of the circle at the point having coordinates σx1= σ avg. and Tx1y1=0.
3. Locate point A, representing the stress conditions on the x face of the element, by plotting its coordinates σx1= σx and Tx1y1= Txy. Note that point A on the circle corresponds to θ=0. Also, note that the x face of the element is labeled "A" to show its correspondence with point A on the circle.
4. Locate point B, representing the stress conditions on the y face of the element, by plotting its coordinates σx1= σy and Tx1y1= -Txy. Note that point B on the circle corresponds to θ=90°. In addition, the y face of the element is labeled "B" to show its correspondence with point B on the circle.
5. Draw a line from point A to point B. This line is a diameter of the circle and passes through the center C. Points A and B, representing the stresses on planes at 90° to each other, are at opposite ends of the diameter (and therefore are 180° apart on the circle).
6. Using point C as the center, draw Mohr's circle through points A and B. The circle drawn in this manner has radius R.

Then, from the geometry of the figure, we obtain the following expressions for the coordinates of point D:





$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + R\cos\beta \qquad \tau_{x_1y_1} = R\sin\beta$$

Mohr Circle Related Problem

Week 15-16

Pages (114-122)

EXAMPLE 3-7

Using Mohr's circle, determine the stresses acting on an element inclined at an angle θ=30°.

Solution:

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = \frac{90 \text{ MPa} + 20 \text{ MPa}}{2} = 55 \text{ MPa}$$

Point *A*, representing the stresses on the *x* face of the element ($\theta = 0$), has coordinates

$$\sigma_{x_1} = 90 \text{ MPa} \qquad \tau_{x_1y_1} = 0$$

Similarly, the coordinates of point *B*, representing the stresses on the *y* face ($\theta = 90°$), are

$$\sigma_{x_1} = 20 \text{ MPa} \qquad \tau_{x_1y_1} = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{90 \text{ MPa} - 20 \text{ MPa}}{2}\right)^2 + 0} = 35 \text{ MPa}$$

Stresses on an element inclined at $\theta = 30°$. The stresses acting on a plane oriented at an angle $\theta = 30°$ are given by the coordinates of point *D*, which is at an angle $2\theta = 60°$ from point *A*

(Point *D*) $\sigma_{x_1} = \sigma_{aver} + R\cos 60°$

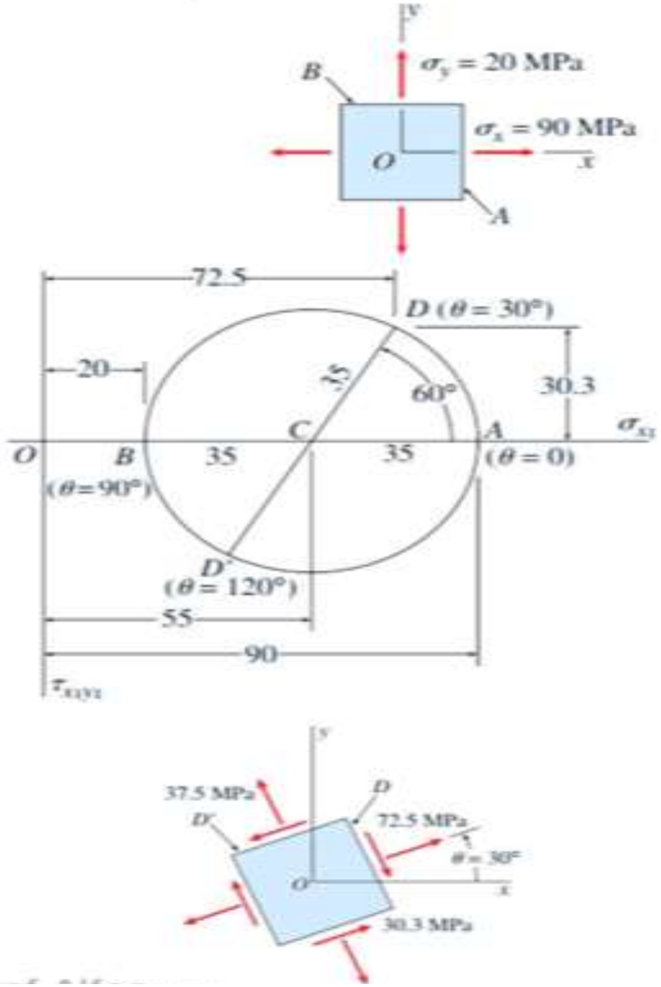
$= 55 \text{ MPa} + (35 \text{ MPa})(\cos 60°) = 72.5 \text{ MPa}$

$\tau_{x_1y_1} = -R\sin 60° = -(35 \text{ MPa})(\sin 60°) = -30.3 \text{ MPa}$

(Point *D'*) $\sigma_{x_1} = \sigma_{aver} - R\cos 60°$

$= 55 \text{ MPa} - (35 \text{ MPa})(\cos 60°) = 37.5 \text{ MPa}$

$\tau_{x_1y_1} = R\sin 60° = (35 \text{ MPa})(\sin 60°) = 30.3 \text{ MPa}$



EXAMPLE 3-8

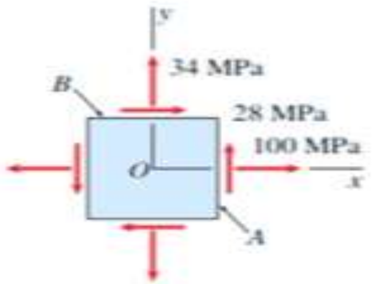
Using Mohr's circle, determine the following quantities:

(a) The stresses acting on an element inclined at an angle θ=40°

(b) The principal stresses

(c) The maximum shear stresses.

(Show all results on sketches of properly oriented elements).



Solution:

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = \frac{100\ \text{MPa} + 34\ \text{MPa}}{2} = 67\ \text{MPa}$$

Point *A*, representing the stresses on the *x* face of the element ($\theta = 0$), has coordinates

$$\sigma_{x_1} = 100\ \text{MPa} \qquad \tau_{x_1y_1} = 28\ \text{MPa}$$

Similarly, the coordinates of point *B*, representing the stresses on the *y* face ($\theta = 90°$) are

$$\sigma_{x_1} = 34\ \text{MPa} \qquad \tau_{x_1y_1} = -28\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{100\ \text{MPa} - 34\ \text{MPa}}{2}\right)^2 + (28\ \text{MPa})^2} = 43\ \text{MPa}$$

$$\tan \overline{ACP_1} = \frac{28\ \text{MPa}}{33\ \text{MPa}} = 0.848 \qquad \overline{ACP_1} = 40.3°$$

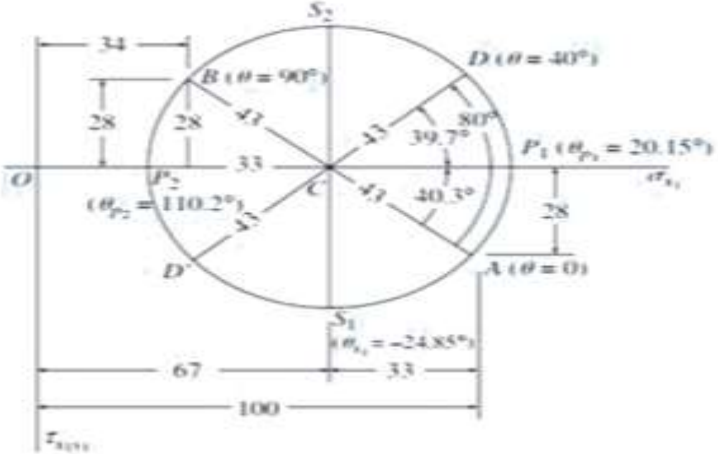
$$\overline{DCP_1} = 80° - \overline{ACP_1} = 80° - 40.3° = 39.7°$$

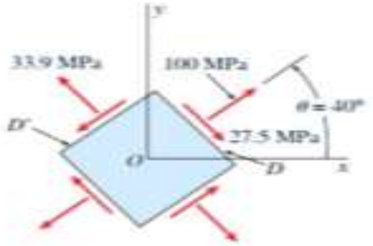
(Point *D*) $\sigma_{x_1} = 67\ \text{MPa} + (43\ \text{MPa})(\cos 39.7°) = 100\ \text{MPa}$

$\tau_{x_1y_1} = -(43\ \text{MPa})(\sin 39.7°) = -27.5\ \text{MPa}$

(Point *D'*) $\sigma_{x_1} = 67\ \text{MPa} - (43\ \text{MPa})(\cos 39.7°) = 33.9\ \text{MPa}$

$\tau_{x_1y_1} = (43\ \text{MPa})(\sin 39.7°) = 27.5\ \text{MPa}$





(b) *Principal stresses*. The principal stresses are represented by points P_1 and P_2 on Mohr's circle . The algebraically larger principal stress (point P_1) is

$$\sigma_1 = 67\ \text{MPa} + 43\ \text{MPa} = 110\ \text{MPa}$$

as seen by inspection of the circle. The angle $2\theta_{p_1}$ to point P_1 from point A is the angle ACP_1 on the circle, that is,

$$\overline{ACP_1} = 2\theta_{p_1} = 40.3° \qquad \theta_{p_1} = 20.15°$$

The algebraically smaller principal stress (represented by point P_2) is obtained from the circle in a similar manner:

$$\sigma_2 = 67\ \text{MPa} - 43\ \text{MPa} = 24\ \text{MPa}$$

The angle $2\theta_{p_2}$ to point P_2 on the circle is $40.3° + 180° = 220.3°$; thus, the second principal plane is defined by the angle $\theta_{p_2} = 110.2°$.

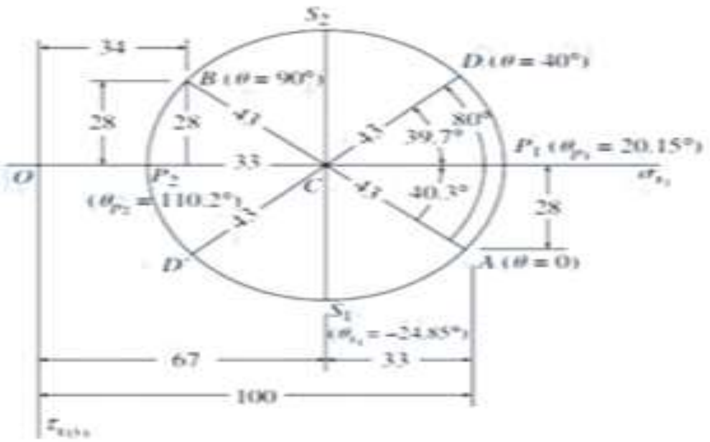
(c) *Maximum shear stresses*. The maximum shear stresses are represented by points S_1 and S_2 on Mohr's circle; therefore, the maximum in-plane shear stress (equal to the radius of the circle) is

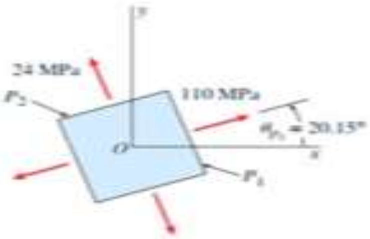
$$\tau_{max} = 43\ \text{MPa}$$

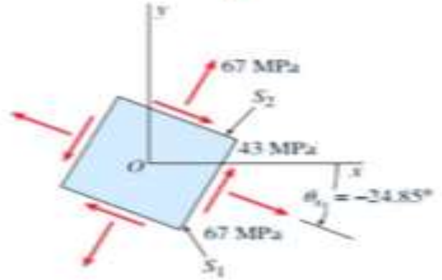
The angle ACS_1 from point A to point S_1 is $90° - 40.3° = 49.7°$, and therefore the angle $2\theta_{s_1}$ for point S_1 is

$$2\theta_{s_1} = -49.7°$$

This angle is negative because it is measured clockwise on the circle. The corresponding angle θ_{s_1} to the plane of the maximum positive shear stress is one-half that value, or $\theta_{s_1} = -24.85°$.
maximum negative shear stress (point S_2 on the circle) has the same numerical value as the maximum positive stress (43 MPa).







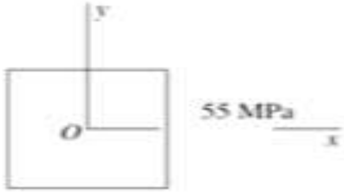
EXAMPLE 3-9

Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at an angle θ=-30° from the x axis (minus means clockwise).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



Solution:

$\sigma_x = 55$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0$

(a) ELEMENT AT $\theta = -30°$ (All stresses in MPa)

$2\theta = -60° \qquad \theta = -30° \qquad R = 27.5$ MPa

Point C: $\sigma_{x_1} = 27.5$ MPa

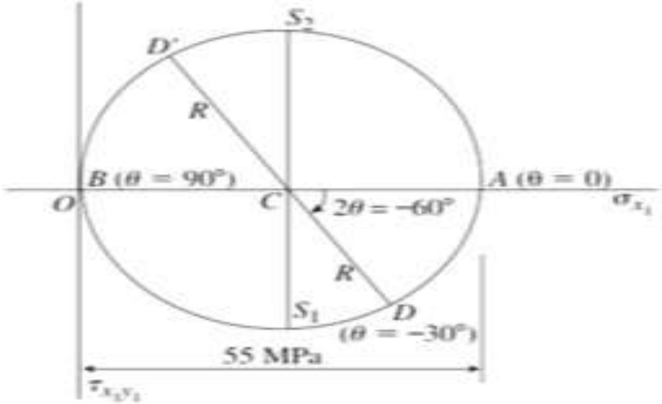
Point D: $\sigma_{x_1} = R + R\cos|2\theta|$

$= R(1 + \cos 60°) = 41.2$ MPa

$\tau_{x_1y_1} = R\sin|2\theta| = R\sin 60° = 23.8$ MPa

Point D': $\sigma_{x_1} = R - R\cos|2\theta| = 13.8$ MPa

$\tau_{x_1y_1} = -R\sin|2\theta| = -23.8$ MPa



(b) MAXIMUM SHEAR STRESSES

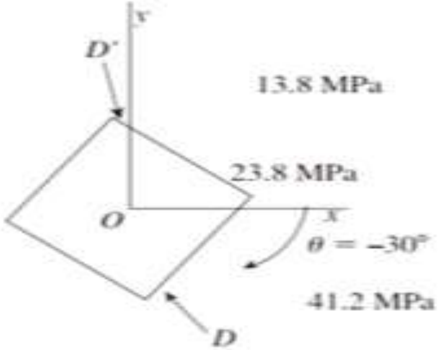
Point S_1: $2\theta_{s_1} = -90° \qquad \theta_{s_1} = -45°$

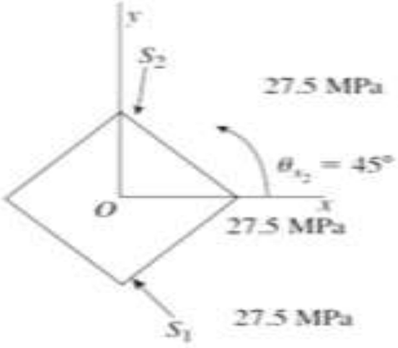
$\tau_{max} = R = 27.5$ MPa

Point S_2: $2\theta_{s_2} = 90° \qquad \theta_{s_2} = 45°$

$\tau_{min} = -R = -27.5$ MPa

$\sigma_{aver} = R = 27.5$ MPa





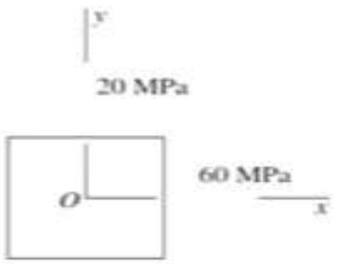
EXAMPLE 3-10

Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at a counterclockwise angle θ=22.5° from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.



Solution:

$\sigma_x = -60$ MPa $\quad \sigma_y = 20$ MPa $\quad \tau_{xy} = 0$

(a) Element at $\theta = 22.5°$

(All stresses in MPa)

$2\theta = 45° \quad \theta = 22.5°$

$2R = 60 + 20 = 80$ MPa $\quad R = 40$ MPa

Point C: $\sigma_{x_1} = -20$ MPa

(b) Maximum shear stresses

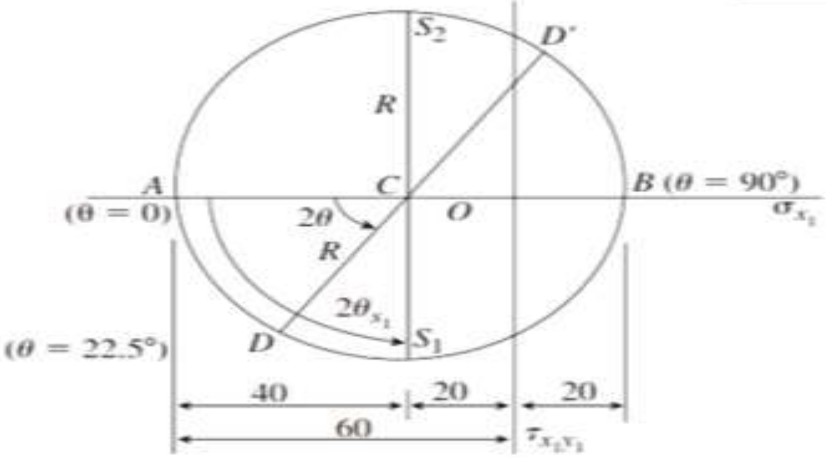
Point S_1: $2\theta_{s_1} = 90° \quad \theta_{s_1} = 45°$

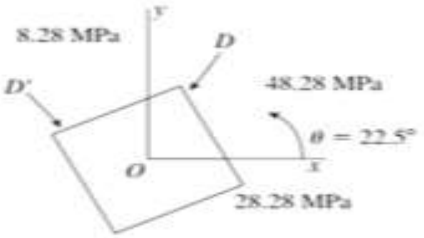
$\tau_{max} = R = 40$ MPa

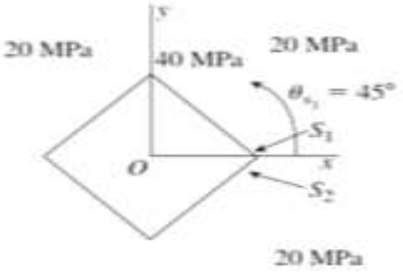
Point S_2: $2\theta_{s_2} = -90° \quad \theta_{s_2} = -45°$

$\tau_{min} = -R = -40$ MPa

$\sigma_{aver} = -20$ MPa







EXAMPLE 3-11

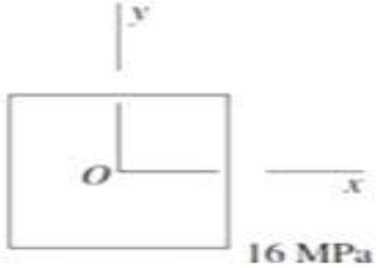
Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at a counterclockwise angle θ=20° from the x axis.

(b) The principal stresses.

Show all results on sketches of properly oriented elements.

Solution:



$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -16 \text{ MPa}$

(a) ELEMENT AT $\theta = 20°$

(All stresses in MPa)

$2\theta = 40° \quad \theta = 20° \quad R = 16 \text{ MPa}$

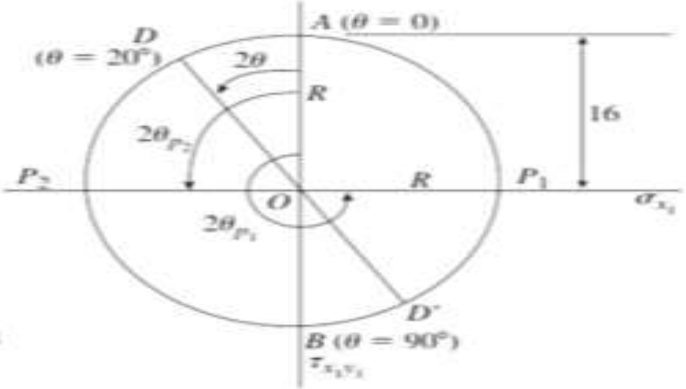
Origin O is at center of circle.

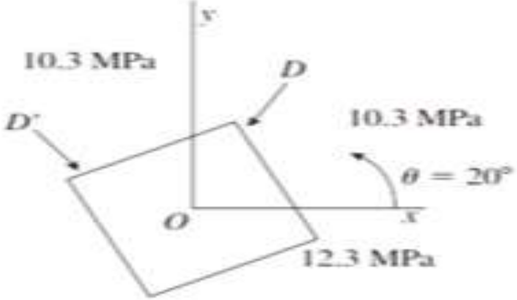
Point D: $\sigma_{x_1} = -R \sin 2\theta = -10.28 \text{ MPa}$

$\tau_{x_1y_1} = -R \cos 2\theta = -12.26 \text{ MPa}$

Point D': $\sigma_{x_1} = R \sin 2\theta = 10.28 \text{ MPa}$

$\tau_{x_1y_1} = R \cos 2\theta = 12.26 \text{ MPa}$





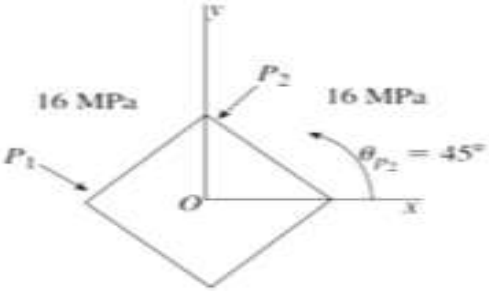
(b) PRINCIPAL STRESSES

Point P_1: $2\theta_{p_1} = 270° \quad \theta_{p_1} = 135°$

$\sigma_1 = R = 16 \text{ MPa}$

Point P_2: $2\theta_{p_2} = 90° \quad \theta_{p_2} = 45°$

$\sigma_2 = -R = -16 \text{ MPa}$



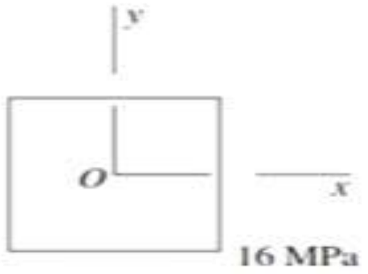
EXAMPLE 3-11

Using Mohr's circle, determine:

(a) The stresses acting on an element oriented at a counterclockwise angle θ=20° from the x axis.

(b) The principal stresses.

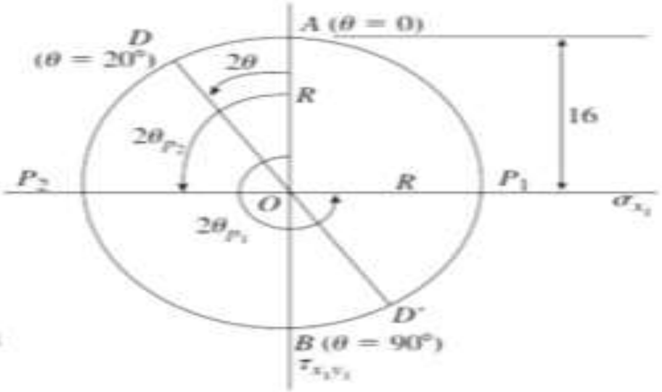
Show all results on sketches of properly oriented elements.

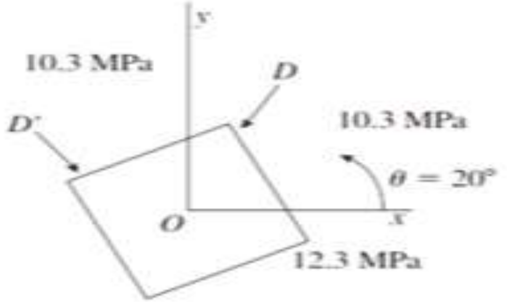


Solution:

$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -16$ MPa

(a) ELEMENT AT $\theta = 20°$





(All stresses in MPa)

$2\theta = 40° \quad \theta = 20° \quad R = 16$ MPa

Origin O is at center of circle.

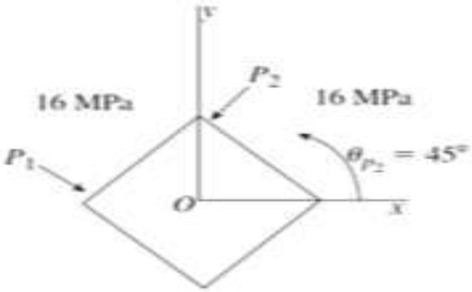
Point D: $\sigma_{x_1} = -R \sin 2\theta = -10.28$ MPa

$\tau_{x_1y_1} = -R \cos 2\theta = -12.26$ MPa

Point D': $\sigma_{x_1} = R \sin 2\theta = 10.28$ MPa

$\tau_{x_1y_1} = R \cos 2\theta = 12.26$ MPa

(b) PRINCIPAL STRESSES



Point P_1: $2\theta_{p_1} = 270° \quad \theta_{p_1} = 135°$

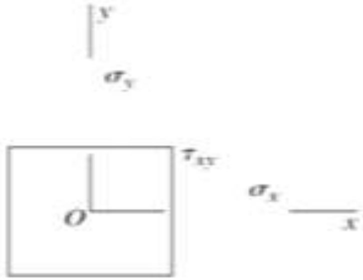
$\sigma_1 = R = 16$ MPa

Point P_2: $2\theta_{p_2} = 90° \quad \theta_{p_2} = 45°$

$\sigma_2 = -R = -16$ MPa

EXAMPLE 3-12

Using Mohr's circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ. ($\sigma_x = 21$ MPa, $\sigma_y = 11$ MPa, $\tau_{xy} = 8$ MPa, $\theta = 50°$)



Solution:

$\sigma_x = 21$ MPa $\quad \sigma_y = 11$ MPa

$\tau_{xy} = 8$ MPa $\quad \theta = 50°$

(All stresses in MPa)

$$R = \sqrt{(5)^2 + (8)^2} = 9.4340 \text{ MPa}$$

$$\alpha = \arctan \frac{8}{5} = 57.99°$$

$$\beta = 2\theta - \alpha = 100° - \alpha = 42.01°$$

Point D $(\theta = 50°)$:

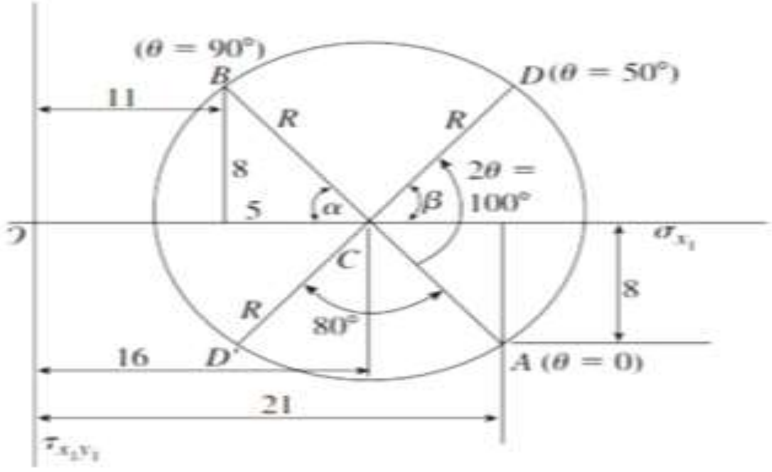
$$\sigma_{x_1} = 16 + R\cos\beta = 23.01 \text{ MPa}$$

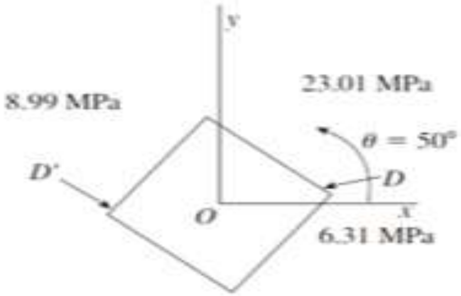
$$\tau_{x_1y_1} = -R\sin\beta = -6.31 \text{ MPa}$$

Point D' $(\theta = -40°)$:

$$\sigma_{x_1} = 16 - R\cos\beta = 8.99 \text{ MPa}$$

$$\tau_{x_1y_1} = R\sin\beta = 6.31 \text{ MPa}$$





H.W

Using Mohr's circle, determine the stresses acting on an element oriented at an angle θ from the x axis. Show these stresses on a sketch of an element oriented at the angle θ.

1- $\sigma_x = -44$ MPa $\quad \sigma_y = -194$ MPa
$\tau_{xy} = -36$ MPa $\quad \theta = -35°$

2- $\sigma_x = 31$ MPa $\quad \sigma_y = -5$ MPa
$\tau_{xy} = 33$ MPa $\quad \theta = 45°$

Thank You